# 1.4 Properties of Exponents

An **exponent** a number that tells us how many times a quantity is multiplied by itself. Another word for exponent is **power**. The quantity that is being multiplied by itself is called the **base**.

$5 \cdot 5 \cdot 5$ Exponent : 3 Base : 5

Ex: $5^3 =$ ~~NOT 5·3~~

$7^2 =$ Exponent 2 Base : 7 $7 \cdot 7$

$21^4 =$ Exponent 4 Base : 21 $21 \cdot 21 \cdot 21 \cdot 21$

$2x^3$ Exponent 3 Base : X Coefficient : 2

Using this information, see if you can figure out some shortcuts or rules for simplifying exponents. Be sure to show your work to help you.

Product of Powers

Simplify the following exponents: Add

$6^3 \cdot 6^5 =$ $6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$ $\boxed{6^8}$

$4^7 \cdot 4^2 =$ $4^{7+2} = \boxed{4^9}$

$12^4 \cdot 12^4 =$ $\boxed{12^8}$

Quotient of Powers

Simplify the following exponents: SUBTRACT

$\frac{2^7}{2^4} = \frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 2 \cdot 2 \cdot 2}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2}}$

$2^{7-4} = 2^3$ ✓ $\boxed{2^3}$ ✓

$\frac{8^5}{8^2} = 8^{5-2} = \boxed{8^3}$

$\frac{10^9}{10^4} = \boxed{10^5}$

Power of Powers

Simplify the following exponents: MULTIPLY

$(2^3)^3 = (2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)$ $\boxed{2^9}$ ✓

$2^{3 \cdot 3} = 2^9$ ✓

$(5^2)^4 =$ $5^{2 \cdot 4} = \boxed{5^8}$

$(8^4)^3 = \boxed{8^{12}}$

| Property | Notation | Rule | Examples |
|---|---|---|---|
| Product of Powers | $x^m \bullet x^n$ | Exponents - ADD<br>Coefficients - Multiply | $a^4 \cdot a^3 = a^{4+3} = a^7$<br>$5x^2 \cdot 2x^9 = 5 \cdot 2 \; x^{2+9}$ $\boxed{10x^{11}}$ |
| Quotient of Powers | $\frac{x^m}{x^n}$ | Exponents - Subtract<br>Coefficients - Divide | $\frac{a^7}{a^2} = a^{7-2} = a^5$<br>$\frac{6x^{10}}{2x^{-1}} = 6 \div 2 \; x^{10--1}$ $3x^{10+1}$ $\boxed{3x^{11}}$ |
| Power of Powers | $(x^m)^n$ | Exponents - Multiply<br>Coefficient - Raise to the power | $(a^3)^2 = a^{3 \cdot 2} = a^6$<br>$(2x^2)^5 = 2^5 x^{2 \cdot 5}$ $\boxed{32x^{10}}$ |

Notice that to simplify the exponents, the two expressions **must** have the same base. If you have coefficients in front of your exponent-base pair, multiply them as normal.

There are two other exponent properties that we need to talk about:

| Property | Rule | Example |
|---|---|---|
| Zero Exponent Property | Anything with exponent of 0 becomes 1 | $a^0 = 1$    $12^0 = 1$ |
| Negative Exponent Property | Negative exponent means the base is in the wrong spot of a fraction. To make the exponent positive, move spots in the fraction. * exponent <u>only</u> applies to what it is attached to | $\frac{x^{-5}}{1} = \frac{1}{x^5}$ <br> $2a^{-2} = \frac{2}{a^2}$ <br> $\frac{5}{3m^{-6}} = \frac{5m^6}{3}$ |

Now we're going to put everything together. **Make sure to remember the order of operations!**

1) Simplify. Your answer should only contain positive exponents.

a. $(2b^4)^0 = 1$

b. $(2b)^4$

$2^4 b^4$

$\boxed{16b^4}$

c. $(2m^2 \cdot 3m)^2$

$(6m^3)^2$

$6^2 m^6 = \boxed{36m^6}$

d. $\frac{(b^4)^2}{b^5}$

$\frac{b^8}{b^5} = \boxed{b^3}$

e. $\frac{5a^7}{4a^4}$

$\frac{5a^3}{4}$

f. $(3x^2)^{-3}$

$3^{-3}x^{-6} = \frac{1}{3^3 x^6} = \boxed{\frac{1}{27x^6}}$

g. $\frac{6y^4 \cdot 3y^2}{2y^3}$

$\frac{18y^6}{2y^3} = \boxed{9y^3}$

h. $\frac{6x^{-2}}{2x^4}$

$\frac{6}{2x^4x^2} = \frac{6}{2x^6} = \boxed{\frac{3}{x^6}}$

i. $\frac{(3m^3)^2}{2m^{-4}}$

$\frac{3^2m^6}{2m^{-4}} = \frac{9m^6m^4}{2}$

$= \boxed{\frac{9m^{10}}{2}}$

HW 3) $\frac{2 \cdot 2^2}{(2^4)^{-4}}$

$2 \cdot 2 \cdot 2$

$= \frac{2^3}{2^{-16}} = 2^3 \cdot 2^{16} = \boxed{2^{19}}$

Sometimes we see an exponent in the form of a fraction. This means that we can rewrite it into a **radical expression**. The denominator of the fraction signifies what kind of root you are taking. The numerator signifies the exponent of the base inside the root.

Ex: $\boxed{3^{\frac{1}{2}} = \sqrt{3}}$     $m^{\frac{1}{5}} = \sqrt[5]{m}$     $y^{\frac{2}{3}} = \sqrt[3]{y^2}$     $2x^{\frac{2}{5}} = 2\sqrt[5]{x^2}$

<u>index</u>, kind of root → $\sqrt[\square]{x^{\square}}$ ← power of base

vs. $x^{\frac{\square}{\square}}$ ← power of base (numerator), ← index (denominator)