2.1 Classifying Numbers

$5^2 =$ $(-4)^2 =$ $\sqrt{64} =$ $\sqrt{-100} =$

It is impossible to take the square root of a negative number; it doesn't exist. That is why these numbers are called ____________________ numbers.

$i =$

In order to find the square root of negative numbers, ____

____.

1) Find each root.

a) $\sqrt{-25}$ b) $\sqrt{-81}$ c) $\sqrt{-121}$

d) $\sqrt{-16}$ e) $\sqrt{-100}$ f) $\sqrt{-169}$

The reason we will care so much about imaginary numbers in this class is that it will assure that every problem has a solution. The most common application outside of this class is for calculations with electricity.

Imaginary numbers are a smaller group of the **complex** numbers. Complex numbers are defined as having a ________ part and an __________________ part.

Complex number:

2) Identify the real and imaginary part of the following complex numbers:

a) $6 + 5i$ b) $8 - 3i$ c) $-4 - 7i$

When classifying numbers, you want to make sure to ____________ the number first.

3) Name the set or sets that each number belongs to. Circle the most specific set:

a) $\sqrt{81}$

b) $\frac{0}{-2}$

c) $\sqrt{\frac{279}{3}}$

d) $\sqrt{225}$

e) $\frac{176}{64}$

f) $\frac{68}{40}$

g) -9+2

h) π+3

<u>Test your understanding</u>

Wh etermine if each statement is always, sometimes, or never true:
of

a. If a number is rational, it can be irrational too.

b. An integer is a whole number.

c. A natural number is a real number.

d. A whole number is a natural number.

Every number can be classified based on its characteristics. Below we are going to draw a nesting model for number classification.

Set	Symbol	Description
Complex		
Real		
Imaginary		
Rational		
Irrational		
Integer		
Whole		
Natural		

2.2 Simplifying Radicals

A **radical** is an expression that involves a square root, cube root, etc. Not all numbers come out with a perfect root, but when we break down a radical, you may find that there are some numbers that can simplify out of the radical.

1) Simplify each radical.

Steps to Simplifying Radicals	a) $\sqrt{63}$

b) $\sqrt{392}$

c) $3\sqrt{112}$

d) $\sqrt[3]{56}$

f) $\sqrt[3]{-132}$

g) $4\sqrt[3]{500}$

h) $\sqrt[4]{405}$

You will also be asked to simplify radicals with imaginary roots. To do this, ______________________________

____.

2) Simplify each radical.

a) $\sqrt{-20}$

b) $\sqrt{-80}$

c) $\sqrt{-120}$

d) $\sqrt{-16}$

e) $\sqrt{-1000}$

f) $\sqrt{-50}$

If you encounter radicals with variables, you will still break it down like normal; however, the variables that come out of the radical may need absolute value signs around them.

When you are asked to find a root, you are being asked for the **principal root**, meaning the positive outcome of the square root. Because the variable might be negative, we need absolute value signs to assure that we are answering with the positive root.

Variables outside the radical need absolute value when:

b) $\sqrt{24x^4y^2}$

c) $\sqrt{12x^3}$

d) $\sqrt{128x^4}$

e) $2\sqrt{175x^3y^2}$

f) $\sqrt[3]{-72x^4}$

The complex number system assumes that there WILL be negatives inside of the radical. Because of this – absolute value signs are NOT necessary when solving the problem!

g) $\sqrt{-25x^2y^4}$

h) $\sqrt{-60xy^3}$

i) $\sqrt{-27x^{10}y^7}$

2.3 Operations in the Number Systems

Rational Vs. Rational	
Add	**Multiply**
$8 + 7 = 15$ $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ Rational	$8 \cdot 7 = 56$ $\frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$ Rational

Rational Vs. Irrational	
Add	**Multiply**
$3.14... + 2 = 5.14...$ Irrational	$2.168... \cdot 5 = 10.168...$ $\pi \cdot 5 = 5\pi$ Irrational

BIG IDEAS

$Q + Q = Q$

$Q(Q) = Q$

$Q + I = I$

$Q(I) = I$

$I + I = I$ OR R

$I(I) = I$ OR R

Q = Rational

I = Irrational

Irrational needs to be written out on the test!

Irrational Vs. Irrational	
Add	**Multiply**
$\pi + \pi = 2\pi$ $\sqrt{3} + \sqrt{3} = 2\sqrt{3}$ IRRATIONAL $\pi - \pi = 0$ $-\sqrt{3} + \sqrt{3} = 0$ Rational	$\pi \cdot \pi = \pi^2$ $\sqrt{2} \cdot \sqrt{3} = \sqrt{6}$ Irrational $\sqrt{3} \cdot \sqrt{3} = \sqrt{9} = 3$ $\sqrt{2} \cdot \sqrt{8} = \sqrt{16} = 4$ (3 3; 4 4) RATIONAL

*Pro tip: type it into your calculator and see what you get

1) Determine whether each sum or product will be rational or irrational.

a. $\sqrt{12} + 6$

Irrational

b. $9 - \frac{2}{3}$

Q or Rational

c. $-\sqrt{5} * \sqrt{20}$ $\quad -\sqrt{100} = 10$

Q

d. $2\pi * \pi$

$2\pi^2$

Irrational

Operations with Complex Numbers

What is a complex number? An expression with Real and Imaginary Parts.

In order to add or subtract complex numbers, we need to Combine Like Terms.

* order the real # 1st + imaginary 2nd

2) Simplify.

a) $(-3 - 9i) + (11 - 71)$

$-63 - 9i$

b) $(-1 - 3i) - (3 - 6i)$

$-1 - 3i - 3 + 6i$

$-4 + 3i$

c) $(6 - 11i) - (11 - 6i)$

$6 - 11i - 11 + 6i$

$-5 - 5i$

We can multiply complex numbers through distribution, but there is something special that happens when we do this. Recall that we learned that $i = \sqrt{-1}$. What happens when we want to find i^2?

$i = \sqrt{-1}$

$i^2 = \sqrt{-1}^2 = -1$

3) Multiply. *Always write real then imaginary*

a) $(-7 + 2i)(-7 - 4i)$

$49 + 28i - 14i - 8i^2$

$-8(-1)$

$49 + 28i - 14i + 8 = 57 + 14i$

b) $(6 + 8i)(-6 - 2i)$

$-36 - 12i - 48i - 16i^2$

$-16(-1)$

$-36 - 12i - 48i + 16 = -20 - 60i$

c) $(4 - 2i)^2 = (4 - 2i)(4 - 2i)$

$16 - 8i - 8i + 4i^2$

-4

$12 - 16i$

d) $(-1 + 6i)(-5 + 4i)$

( ) + ( )

Combine like terms

( )( )

Distribute

g) $5(2 - 3i) + 4(2i)$

$10 - 15i + 8i$

$10 - 7i$

h) $(3 + 9i) - 5(2 - i)$

$3 + 9i - 10 + 5i$

$-7 + 14i$