Intersection - box where two categories meet

Union - two totals subtract the intersection (otherwise intersection would be counted twice)

9.2 Conditional Probability

A **two-way frequency table** is a way to represent data that is described by multiple categories. The numbers that are in the total rows are called marginal (single category) frequencies. The other numbers in the table are called joint (two categories) frequencies.

1) Fill out the table below as a class. Then calculate each probability.

	Black and Blue	White and Gold	Total
Boys	6	4	10
Girls	9	5	14
Total	15	9	24

a) P(Black and Blue)

$\frac{15}{24}$, 62.5%

b) P(White and Gold)

$\frac{9}{24}$, 37.5%

c) P(Boy)

$\frac{10}{24} = \boxed{\frac{5}{12}, 41.7\%}$

d) P(Girl)

$\frac{14}{24} = \boxed{\frac{7}{12}, 58.3\%}$

e) P(Boy ∩ Black and Blue) (intersection)

$\frac{6}{24} = \boxed{\frac{1}{4}, 25\%}$

f) P(Girl ∩ White and Gold)

$\frac{5}{24}$, 20.8%

g) P(Boy ∪ White and Gold) (union)

Boy + WtG − BnWtG

10 + 9 − 4 = 15

$\frac{15}{24} = \boxed{\frac{5}{8}}$

h) P(Girl ∪ Black and Blue)

Girl + B&B − Girl ∩ B&B

14 + 15 − 9 = 20

$\frac{20}{24} = \boxed{\frac{5}{6}, 83.3\%}$

Two-way tables help us calculate what we call **conditional probability**, meaning the probability that an event happens given the fact that something else has already happened.

P(A|B): Probability of A given B

P(B|A): Probability of B given A

* Second event listed becomes the new total category; it is the only row/column of the table that you look at

2) Use the table above to find each probability.

a) P(Black and Blue | Boy) (Boy = Total)

Boys who see B&B → $\frac{6}{10} = \boxed{\frac{3}{5}, 60\%}$ ← Boys

b) P(Girl | Black and Blue) (Black and Blue = Total)

$\frac{9}{15} = \boxed{\frac{3}{5}, 60\%}$

c) P(White and Gold | Girl) (Girl = Total)

$\frac{5}{14}$, 35.7%

1 category: marginal, use margins of table

3) The Math Club is going to sell candy as a fundraiser. They surveyed 80 students about their candy preferences. Some of the results are shown below. Fill in the missing information.

	Chocolate	White Chocolate	Total
Twizzlers	19	18	37
Redvines	26	17	43
Total	45	35	80

Use the table to find the given probabilities.

a) P(Chocolate)

$\frac{45}{80} = \frac{9}{16}$, 56.3%

b) P(White Chocolate)

$\frac{35}{80} = \frac{7}{16}$, 43.8%

c) P(Twizzlers)

$\frac{37}{80}$, 46.3%

d) P(Redvines)

$\frac{43}{80}$, 54%

e) P(Chocolate ∩ Redvines)

Intersection

$\frac{26}{80} = \frac{13}{40}$, 32.5%

f) P(White Chocolate ∩ Twizzlers)

$\frac{18}{80} = \frac{9}{40}$, 22.5%

g) P(Chocolate ∪ Twizzlers)

C + T − C∩T

45 + 37 − 19 = 63

$\frac{63}{80}$, 78.8%

h) P(White Chocolate ∪ Redvines)

WC + R − WC∩R

35 + 43 − 17 = 61

$\frac{61}{80}$, 76.3%

i) P(Twizzlers | Chocolate)

$\frac{\text{Twizzlers}}{\text{Chocolate}} = \frac{19}{45}$, 42.2%

Twizzlers given chocolate, meaning chocolate column is only column you're looking at

j) P(Chocolate | Redvines)

$\frac{C}{R} = \frac{26}{43}$, 60.5%

4) Use the table below to calculate each probability.

GPA	< 2.0	2.0-3.0	> 3.0	Total
Many Skipped Classes	80	25	5	110
Few Skipped Classes	175	450	265	890
Total	255	475	270	1000

a) What is the probability a student has a GPA between 2.0 and 3.0?

$\frac{475}{1000} = \frac{19}{40}$, 47.5%

b) What is the probability that a student has a GPA under 2.0 and has skipped many classes?

Intersection

$\frac{80}{1000} = \frac{2}{25}$, 8%

c) What is the probability that a student has a GPA under 2.0 or has skipped many classes?

Union

Under 2.0 + skipped − intersection

255 + 110 − 80 = 285

$\frac{285}{1000} = \frac{57}{200}$, 28.5%

d) What is the probability that the student's GPA is under 2.0 and they did not skip many classes.

Intersection

$\frac{175}{1000} = \frac{7}{40}$, 17.5%

e) What is the probability that a student has a GPA under 2.0 given the fact that they skipped many classes?

Conditional

Total category

$\frac{80}{110} = \frac{8}{11}$, 72.7%

f) What is the probability that a student skipped few classes given the fact that they have a FPA between 2.0 and 3.0?

Total category

$\frac{450}{475} = \frac{18}{19}$, 94.7%