8.2 Exponential Functions and Average Rate of Change

$y = a \cdot b^x$

a: Initial value (y-int)

b: Growth/decay factor

Exponential Functions

What is different about an exponential function? The variable is in the exponent.

We can have equations that either represent exponential growth or exponential decay. Let's see what happens when we graph:

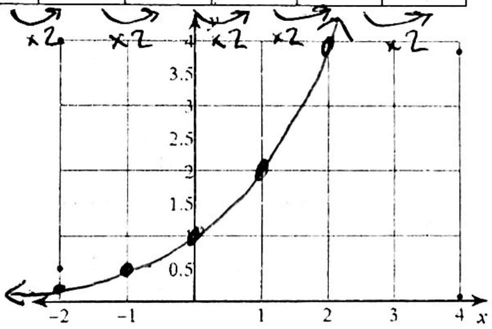
To determine what makes our b value a growth factor or a decay factor, find each of the following

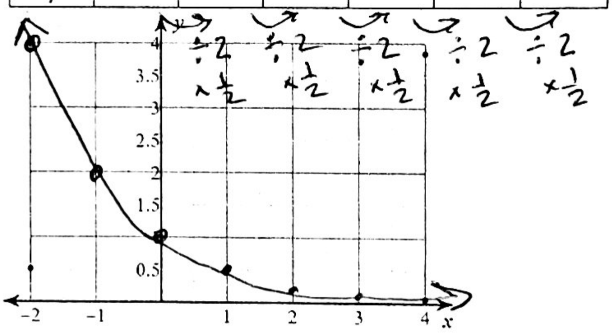
$f(x) = 2^x$ $a = 1$ $b = 2$

x	-2	-1	0	1	~~3~~ 2	~~4~~ 3
y	.25	.5	1	2	4	8

$g(x) = \left(\frac{1}{2}\right)^x$ $a = 1$ $b = \frac{1}{2}$

x	-2	-1	0	1	~~3~~ 2	~~4~~ 3
y	4	2	1	.5	.25	.13





Growth Factor	Decay factor
$b > 1$	$b < 1$

1) Determine the initial value and the growth or decay factor for the equations below.

a. $y = 60(1.41)^x$

IV: 60

GF: 1.41

b. $y = 23(0.67)^x$

IV: 23

DF: 0.67

c. $y = 525(0.86)^x$

IV: 525

DF: 0.86

Another way to think about the growth/decay factor is to ask yourself, "What value is attached to the exponent?"

The growth/decay **RATE** is the percent change between each output value of the function.

Let's see if you can figure out how to find the growth/decay rate by looking at the following examples:

Growth rate: 23% Growth rate: 64% Decay rate: 25% Decay rate: 61%

$b=1.23$ $f(x)=3(1.\underline{23})^{x}$ $g(x)=0.5(1.\underline{64})^{x}$ $h(x)=2(\frac{3}{4})^{x}$ $k(x)=0.4(0.39)^{x}$

Growth Rate	Decay rate
$b-1$	$1-b$

How far is the growth/decay factor from 1?

2) Identify the initial value, the growth or decay factor, and the growth or decay rate for each of the functions below.

a. $f(x)=4(0.78)^{x}$
IV: 4
DF: 0.78 DR: 22%

b. $y=5(1.47)^{x}$
IV: 5
GF: 1.47 GR: 47%

c. $g(t)=0.6(1.19)^{t}$
IV: 0.6
GF: 1.19 GR: 19%

d. $y=1.5(0.36)^{x}$
IV: 1.5
DF: 0.36 DR: 64%

e. $h(x)=3(\frac{2}{5})^{x}$
IV: 3
DF: $\frac{2}{5}=0.4$
DR: 60%

f. $k(x)=2\cdot 2^{x}$
IV: 2
GF: 2 GR: 100%

3) Find the growth or decay rate factor for the functions below and state the growth or decay rate.

a. $f(x)=(1.05^{4})^{x}$
GF: 1.22
GR: 22%

b. $h(t)=(0.68^{3})^{t}$
DF: 0.31
DR: 69%

c. $y=1.46^{3x}$

4) Find a bank account balance if the account starts with \$100, has an annual growth rate of 4%, and the money left in the account for 12 years.

$y=a\cdot b^{x}$ ← usually time

↑ initial ↖ growth/decay factor

$a=100$
GR = 4% = .04
GF = 1.04

$y=100(1.04)^{12}=\boxed{\$160.10}$

→ 5) In 1985, there were 285 cell phone subscribers in the small town of Centerville. The number of subscribers increased by 75% per year after 1985. How many cell phone subscribers were in Centerville in 1994?

$a=285$
$1994-1985=9$
GR: 75% = 0.75
GF: 1.75

$y=285(1.75)^{9}=43871.99$

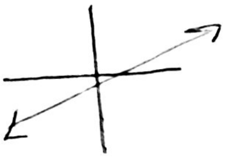
$\boxed{\text{About } 43{,}872}$

6) Each year the local country club sponsors a tennis tournament. Play starts with 128 participants. During each round, half of the players are eliminated. How many players remain after 5 rounds?

Average Rate of Change

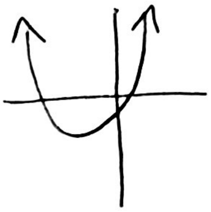
Average Rate of Change

a. Give an example of a function that has a constant rate of change. Give an example of a graph and an equation.



$y = 2x + 1$

b. Give an example of a function that does not have a constant rate of change.



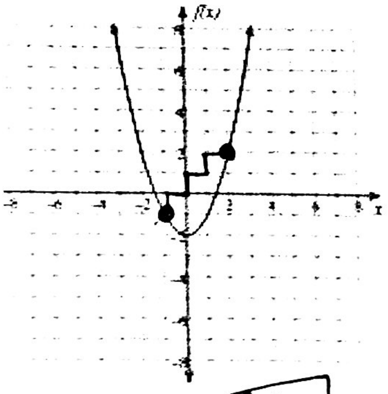
$y = -2x^2 + 7x + 2$

Because not all equations have a constant rate of change, we have to take a look at what we call the **average rate of change.** This looks at the rate of change of a function over a specified interval. These intervals refer to the x-values.

Average Rate of Change Formula	Which also means...
On the interval $[a, b]$, $\frac{f(b) - f(a)}{b - a}$	$\frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$ $\boxed{\text{Slope}}$ $\frac{\text{Rise}}{\text{Run}}$

1) Find the ~~average rate of change~~ slope for each function over the given interval.

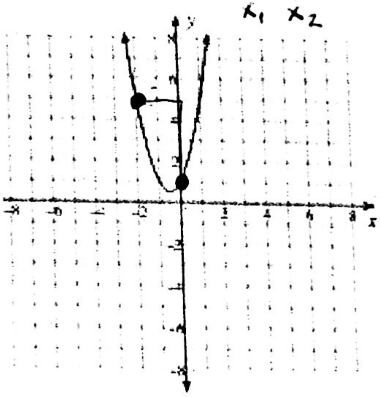
a. $f(x) = x^2 - 2$; $[-1, 2]$



$\boxed{1}$

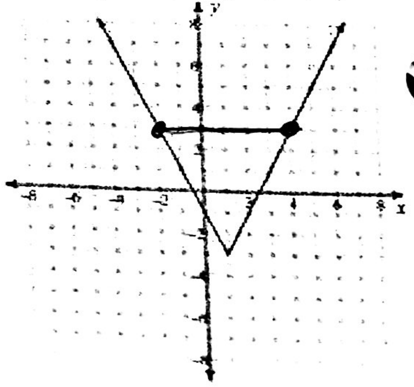
b. $y = 2x^2 + 2x + 1$; $[-2, 0]$

x_1 x_2



$\frac{-4}{2} = \boxed{-2}$

c. $g(x) = 2|x - 1| - 3$; $[-2, 4]$



$\frac{0}{6} = \boxed{0}$

2) Find the slope between the two points.

a. ~~and~~ (2, 5) and (6, 17)

x	y
2	5
6	17

Run +4 Δx

Rise +12 Δy

$\frac{12}{4} = \boxed{3}$

b. ~~And~~ (−7, −4) and (1, 13)

Steps for Average Rate of Change

1) Plug x's into equation to get y's

2) Find the slope between the two points

ex: Find the ~~average rate of change~~ slope for $y = x^2 - 2x + 1$ over the interval [-2, 3].

x_1 x_2

$x_1 = -2$ $y = (-2)^2 - 2(-2) + 1 = 9$ $(-2, 9)$

$x_2 = 3$ $y = (3)^2 - 2(3) + 1 = 4$ $(3, 4)$

Run	x	y	Rise
+5	-2	9	-5
	3	4	

$\frac{-5}{5} = \boxed{-1}$

2) Find the ~~average rate of change~~ slope for each function over the specified interval.

a. over [-1, 6] $y = 5x - 4$

↗ $\boxed{5}$

Line- constant rate of change, slope is 5

b. over [2, 5] $y = -x^2 + 5$

x_1 x_2

$x_1 = 2$ $y = -(2)^2 + 5 = 1$ $(2, 1)$

$x_2 = 5$ $y = -(5)^2 + 5 = -20$ $(5, -20)$

Run	x	y	Rise
+3	2	1	-21
	5	-20	

$\frac{-21}{3} = \boxed{-7}$

c. over [-8, -4] $y = 2|x+7| + 3$

x_1 x_2

$x_1 = -8$ $y = 2|-8+7| + 3$

$= 2|-1| + 3$

$= 2(1) + 3$

$= 5$

d. over [1,4] $y = 2x^2 - 4x + 1$