

## 7.3 Solving Systems by Substitution

1) Solve each system with substitution.

$$\begin{aligned} y &= -2x + 4 \\ -7x - 3y &= -12 \\ -7x - 3(-2x + 4) &= -12 \\ -7x + 6x - 12 &= -12 \\ -x - 12 &= -12 \\ +12 \quad +12 & \\ -x &= 0 \\ x &= 0 \end{aligned}$$

$$\begin{aligned} y &= -2x + 4 \\ y &= -2(0) + 4 \\ &= 0 + 4 = 4 \end{aligned}$$

$$\boxed{(0, 4)}$$

$$\begin{aligned} b) \quad 6x + y &= 16 & 6x + y &= -16 \\ -8x - 3y &= -8 & -6x & -6x \end{aligned}$$

$$\begin{aligned} y &= -6x + 16 \\ -8x - 3(-6x + 16) &= -8 \\ -8x + 18x - 48 &= -8 \end{aligned}$$

$$10x - 48 = -8$$

$$\begin{aligned} \frac{10x}{10} &= \frac{40}{10} \\ x &= 4 \end{aligned}$$

$$\begin{aligned} x &= 4 \\ y &= -6x + 16 \\ y &= -6(4) + 16 \\ &= -24 + 16 \\ &= -8 \end{aligned}$$

$$\boxed{(4, -8)}$$

The big idea of substitution: Get a variable by itself, then plug it into the other equation

2) Solve each system with substitution.

$$\begin{aligned} a) \quad x^2 + y^2 &= 68 \\ y &= -4x \end{aligned}$$

$$\begin{aligned} x^2 + (-4x)^2 &= 68 \\ x^2 + 16x^2 &= 68 \end{aligned}$$

$$\frac{17x^2}{17} = \frac{68}{17}$$

$$\sqrt{x^2} = \sqrt{4}$$

$$x = \pm 2$$

Plug in both answers to get 2 solutions

$$y = -4x$$

$$\begin{aligned} x = 2 \quad y &= -4(2) \\ &= -8 \end{aligned}$$

$$\boxed{(2, -8)}$$

$$\begin{aligned} x = -2 \quad y &= -4(-2) \\ &= 8 \end{aligned}$$

$$\boxed{(-2, 8)}$$

$$\begin{aligned} c) \quad y &= x^2 - 11x + 28 \\ y &= -3x + 12 \end{aligned}$$

$$\begin{aligned} b) \quad y &= 2x^2 + 13x + 15 \\ y &= x - 1 \end{aligned}$$

$$\begin{aligned} \text{Get one side} &= 0 \\ x - 1 &= 2x^2 + 13x + 15 \\ -x + 1 & \quad -x \quad +1 \end{aligned}$$

$$0 = 2x^2 + 12x + 16$$

$$0 = 2(x^2 + 6x + 8) \quad \frac{8x^2}{4 \cdot 2}$$

$$\begin{aligned} 0 &= 2(x+4)(x+2) \\ x &= -4, -2 \end{aligned}$$

$$x = -4 \quad y = -4 - 1 = -5 \quad \boxed{(-4, -5)}$$

$$x = -2 \quad y = -2 - 1 = -3 \quad \boxed{(-2, -3)}$$

When both equations are  $y =$ , set them equal to each other

$$\begin{aligned} d) \quad x^2 + y^2 &= 25 \\ y &= 2x - 10 \end{aligned}$$

$$x^2 + (2x - 10)^2 = 25$$

$$x^2 + (2x - 10)(2x - 10) = 25$$

$$x^2 + 4x^2 - 20x - 20x + 100 = 25$$

$$5x^2 - 40x + 75 = 0$$

$$5(x^2 - 8x + 15) = 0 \quad \frac{15x^2}{-5 \cdot -3}$$

$$\begin{aligned} 5(x-5)(x-3) &= 0 \\ x &= 5, 3 \end{aligned}$$

$$x = 5 \quad y = 2(5) - 10 = 10 - 10 = 0 \quad \boxed{(5, 0)}$$

$$x = 3 \quad y = 2(3) - 10 = 6 - 10 = -4 \quad \boxed{(3, -4)}$$

Need  $\pm$  when taking a  $\sqrt{\quad}$

e)  $y = x - 5$   
 $y = (x + 2)(x - 2)$

f)  $y = x - 1$   
 $y = x^2 - 5$   
 $x - 1 = x^2 - 5$   
 $-x + 1 = -x + 1$

This is complicated. Don't worry too much about it.

$0 = x^2 - x + 4$   
 $a=1 \quad b=-1 \quad c=4$

Doesn't factor

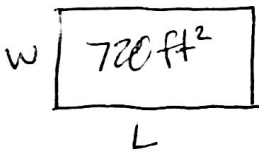
$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(4)}}{2(1)}$   
 $= \frac{1 \pm \sqrt{1 + 16}}{2}$   
 $= \frac{1 \pm \sqrt{17}}{2}$

$x = \frac{1 + \sqrt{17}}{2} \quad y = \frac{1 + \sqrt{17}}{2} - 1$   
 $= \frac{1 + \sqrt{17}}{2} - \frac{2}{2}$   
 $= \frac{1 - 2 + \sqrt{17}}{2} = \frac{-1 + \sqrt{17}}{2}$

$x = \frac{1 - \sqrt{17}}{2} \quad y = \frac{1 - \sqrt{17}}{2} - 1$   
 $= \frac{1 - \sqrt{17}}{2} - \frac{2}{2} = \frac{-1 - \sqrt{17}}{2}$

$\left( \frac{1 + \sqrt{17}}{2}, \frac{-1 + \sqrt{17}}{2} \right) \quad \left( \frac{1 - \sqrt{17}}{2}, \frac{-1 - \sqrt{17}}{2} \right)$

3) You are in a woodshop class and one of your projects is to build an outdoor stage for the drama club's upcoming production. The area of the stage needs to be  $720 \text{ ft}^2$ , and the length has to be 6 feet less than the width. What are the dimensions of the length and width of the stage?



Area:  $L \cdot W = 720$   
 Length is 6 ft less than width:  $L = W - 6$

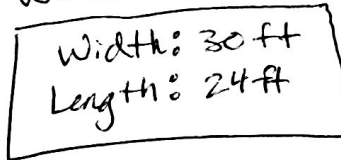
Negative width is not a thing, so  $w = 30$

$(W - 6)W = 720$   
 $W^2 - 6W = 720$   
 $-720 \quad -720$

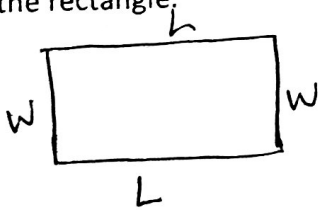
$W = 30 \quad L = 30 - 6 = 24$

$-720W^2$   
 $-30 \wedge 24$

$W^2 - 6W - 720 = 0$   
 $(W - 30)(W + 24) = 0$   
 $W = 30, -24$



4) The perimeter of a rectangle is 42 cm and the area is  $108 \text{ cm}^2$ . Find the dimensions of the length and width of the rectangle.



Area:  $L \cdot W = 108$   
 Perimeter:  $2L + 2W = 42$   
 $-2W \quad -2W$

$(-W + 21)W = 108$

$-W^2 + 21W = 108$   
 $-108 \quad -108$

$W = 12 \quad L = -12 + 21 = 9$   
 $W = 9 \quad L = -9 + 21 = 12$

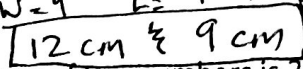
$\frac{2L}{2} = \frac{-2W + 42}{2}$   
 $L = -W + 21$

$-W^2 + 21W - 108 = 0$

$-(W^2 - 21W + 108) = 0$

$-(W - 12)(W - 9) = 0$   
 $W = 12, 9$

$108W^2$   
 $-12 \quad -9$



5) The sum of two numbers is 21. The sum of their squares is 233. Find the values of the two numbers.

Sum:  $x + y = 21$

$x + y = 21$   
 $-x \quad -x$

Sum of squares:  $x^2 + y^2 = 233$

$y = -x + 21$

$x^2 + (-x + 21)^2 = 233$

$2x^2 - 42x + 208 = 0$

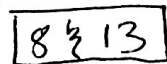
$2(x^2 - 21x + 104) = 0$

$x^2 + (-x + 21)(-x + 21) = 233$

$2(x - 8)(x - 13) = 0$

$x^2 + x^2 - 21x - 21x + 441 = 233$   
 $-233 \quad -233$

$x = 8, 13$



$104$   
 $-8 \quad -13$   
 $8 + y = 21$   
 $y = 13$