4.3 Solving by Completing the Square

Today we are going to look at perfect square trinomials, meaning the trinomials that are a result of a factor being squared.

Example 1: Distribute the following expressions.

a) $(x+5)^2$

$(x+5)(x+5)$

$x^2 + 5x + 5x + 25$

$\boxed{x^2 + 10x + 25}$

b) $(x+3)^2$

$(x+3)(x+3)$

$x^2 + 3x + 3x + 9$

$\boxed{x^2 + 6x + 9}$

c) $(x-7)^2$

$(x-7)(x-7)$

$x^2 - 7x - 7x + 49$

$\boxed{x^2 - 14x + 49}$

Let's look at some patterns going on here: $ax^2 + bx + c$

1) How does the b (middle) coefficient relate to the number in the factor?

It is twice as much as the number in the factor

2) How does the number in the factor relate to the constant, c?

You square the number in the factor to get c $\boxed{c = \left(\frac{b}{2}\right)^2}$

This kind of relationship always exists with a perfect square quadratic. We will use this to do a process called completing the square, which makes any expression into a perfect square quadratic.

Example 2: Determine if the following are perfect square trinomials. Explain your answer.

a) $x^2 + 10x - 11 = 0$

$b = 10 \quad c = -11$

$\left(\frac{10}{2}\right)^2 = (5)^2 = 25$

No, $c \neq 25$

b) $x^2 - 8x + 64 = 0$

$b = -8 \quad c = 64$

$\left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$

No, $c \neq 16$

Example 3: Find the value of c that completes the square.

a) $x^2 - 6x + c$

$b = -6$

$\left(\frac{-6}{2}\right)^2 = (-3)^2 = \boxed{9}$

b) $x^2 + 36x + c$

$b = 36$

$\left(\frac{36}{2}\right)^2 = (18)^2 = \boxed{324}$

c) $x^2 - 20x + c$

$b = -20$

$\left(\frac{-20}{2}\right)^2 = (-10)^2 = \boxed{100}$

Now we are going to practice a process called **completing the square.** The reason that we complete the square is so that we can solve any ~~[illegible]~~ equation by taking a square root.

Steps to Complete the Square:

1) Move constant to other side
2) Find value that completes the square $\left(\frac{b}{2}\right)^2$ and add it to both sides of the equation
3) Rewrite the trinomial as a squared binomial $\left(x+\frac{b}{2}\right)^2$
4) Solve by taking a square root

Example 4: Solve each equation by completing the square.

The easiest time to use complete the square to solve is when $a=1$ and b is even.

a) $x^2 + 8x + 15 = 0$

$-15 \quad -15$

$x^2 + 8x + 16 = -15 + 16$

$\left(\frac{8}{2}\right)^2 = (4)^2 = 16$

$\sqrt{(x+4)^2} = \sqrt{1}$

$x+4 = \pm 1$

$-4 \quad -4$

$x = -4 \pm 1$

$-4+1 = \boxed{-3}$

$-4-1 = \boxed{-5}$

b) $x^2 + 18x + 10 = 0$

$-10 \quad -10$

$x^2 + 18x + 81 = -10 + 81$

$\left(\frac{18}{2}\right)^2 = (9)^2 = 81$

$\sqrt{(x+9)^2} = \sqrt{71}$

$x+9 = \pm\sqrt{71}$

$-9 \quad -9$

$\boxed{x = -9 \pm \sqrt{71}}$

c) $x^2 - 12x - 31 = 0$

$+31 \quad +31$

$x^2 - 12x + 36 = 31 + 36$

$\left(\frac{-12}{2}\right)^2 = (-6)^2 = 36$

$\sqrt{(x-6)^2} = \sqrt{67}$

$x - 6 = \pm\sqrt{67}$

$+6 \quad +6$

$x = \boxed{6 \pm \sqrt{67}}$

d) $x^2 + 6x + 58 = 0$

$-58 \quad -58$

$x^2 + 6x + 9 = -58 + 9$

$\left(\frac{6}{2}\right)^2 = (3)^2 = 9$

$\sqrt{(x+3)^2} = \sqrt{-49}$

$x+3 = \pm 7i$

$-3 \quad -3$

$\boxed{x = -3 \pm 7i}$

When solving, when can you tell if you'll have rational, irrational, or imaginary solutions?

When you simplify the square root