4.1 Solving by Factoring

The Fundamental Theorem of Algebra: The degree (highest exponent) of an equation tells you how many solutions there are.

Ex: $x^{3}-2x+7=0$ has 3 solutions

1) Factor each expression:

a) $p^2-10p+9$

$9p^2$

$-9 \quad -1$

$\boxed{(p-9)(p-1)}$

$a=1$: Shortcut

b) $2b^2-12b-54$

$2(b^2-6b-27)$

$-27b^2$

$-9 \quad 3$

$\boxed{2(b-9)(b+3)}$

GCF

$a=1$: shortcut

c) $\sqrt{n^2}-\sqrt{81}$

Missing middle term: Difference of squares

$\boxed{(n+9)(n-9)}$

d) $2x^2-17x-9$

$-18x^2$

$-18 \quad 1$

$2x^2-18x+1x-9$

$2x(x-9)+1(x-9)$

$\boxed{(2x+1)(x-9)}$

$a\neq1$ Factor by grouping

To introduce solving by factoring, we need to talk about something called the **zero product property**. Here's what it looks like:

a. Solve $3x=0$

What does x equal? How do you know?

$x=0$ since $3\cdot0=0$

b. Solve $3(x+1)=0$

$x+1=0$

$\boxed{x=-1}$

The **zero product property** says that if two terms multiply to be 0, then one of those terms must be 0.

What is a quick way we can see the solution to this equation without taking all the steps to solve?

Find the value that makes the factor 0

If we can break down an equation into its factors, then solving becomes a lot faster using the method you just discovered.

2) Solve each of the following:

a) $p^2-10p+9=0$

$9p^2$

$(p-9)(p-1)=0 \quad -9 \quad -1$

$p-9=0 \quad p-1=0$

$\boxed{p=9} \quad \boxed{p=1}$

b) $\sqrt{n^2}-\sqrt{81}=0$

$(n+9)(n-9)=0$

$n+9=0 \quad n-9=0$

$\boxed{n=-9} \quad \boxed{n=9}$

The big idea behind solving by factoring:

Factor & find the value that makes the factor 0 (take opposite)

Before you begin solving, one side must equal 0 and the equation must be in standard form.

3) Solve each equation.

a) $k^2 + 9k + 14 = 0$ $\quad 14k^2$

$(k+7)(k+2)=0 \quad 7 \; 2$

$\boxed{k=-7}$ $\boxed{k=-2}$

b) $-x^2 - 4x - 4 = 0$

$-(x^2+4x+4)=0$

$-(x+2)(x+2)=0 \quad 4x^2$

$\boxed{x=-2}$ $\boxed{x=-2}$ $\quad 2 \; 2$

Tip #1

A GCF out front does not affect your solutions.

c) $6k^2 + 7k = -2$

$+2 \quad +2$

$6k^2+7k+2=0 \quad 12k^2$

$6k^2+3k+4k+2=0 \quad 4 \; 3$

$3k(2k+1)+2(2k+1)=0$

$(3k+2)(2k+1)=0$

$\boxed{k=\frac{-2}{3}}$ $\boxed{k=-\frac{1}{2}}$

d) $x^2 - 22 = 3$

$-3 \quad -3$

$x^2-25=0$

$(x+5)(x-5)=0$

$\boxed{x=-5}$ $\boxed{x=5}$

Tip #2

$(3x-4)(x-1)=0$

$x=\frac{4}{3} \quad x=1$

Take opposite & divide by coefficient

e) $n^2 - 7n = 0$

$n(n-7)=0$

$\boxed{n=0}$ $\boxed{n=7}$

f) $8x^2 + 2x - 6 = -5$

$+5 \quad +5$

$8x^2+2x-1=0 \quad -8x^2$

$8x^2+4x-2x-1=0 \quad 4 \; -2$

$4x(2x+1)-1(2x+1)=0$

$(4x-1)(2x+1)=0$

$\boxed{x=\frac{1}{4}}$ $\boxed{x=-\frac{1}{2}}$

Tip #3

A variable by itself out front gives a solution of $x=0$

ex: $x(x-5)$

$x=0 \quad x=5$

g) $2x^2 - 17x - 9 = 0$ $\quad -18x^2$

$2x^2-18x+1x-9=0 \quad -18 \; +1$

$2x(x-9)+1(x-9)=0$

$(2x+1)(x-9)=0$

$\boxed{x=-\frac{1}{2}}$ $\boxed{x=9}$

h) $4a^2 + 121 = 0$

$\sqrt{4a^2}-\sqrt{(-121)}=0$ $\quad$ Difference of squares

$(2a+11i)(2a-11i)=0$

$\boxed{a=\frac{-11i}{2}}$ $\boxed{a=\frac{11i}{2}}$