

3.4 Difference of Squares

$$b=0$$

Factor $x^2 - 64$.

$$\boxed{x^2 + 0x - 64}$$

$$\boxed{(x+8)(x-8)}$$

$$\begin{array}{c} -64x^2 \\ 8x \quad \wedge \quad -8x \end{array}$$

Subtraction
Difference of Squares

$$(a^2 - b^2) = (a+b)(a-b)$$

We're going to look at some more expressions like this one. See if you can pick out any patterns.

$$\begin{array}{c} x^2 - 25 \\ x^2 + 0x - 25 \\ \hline (x+5)(x-5) \end{array}$$

$$\begin{array}{c} -25x^2 \\ 5 \quad \wedge \quad -5 \end{array}$$

$$\begin{array}{c} x^2 - 49 \\ \hline (x+7)(x-7) \end{array}$$

$$\begin{array}{c} x^2 - 1 \\ \hline (x+1)(x-1) \end{array}$$

$$\begin{array}{c} x^2 - 81 \\ \hline (x+9)(x-9) \end{array}$$

The binomial in the factor is just the square root of both terms.
Now that we've picked out the pattern, try the questions below.

What do you look for first when you factor? GCF

1) Factor each expression below.

a) $\sqrt{x^2 - 121}$

$$(x+11)(x-11)$$

b) $\sqrt{9x^2 - 25}$

$$(3x+5)(3x-5)$$

c) $2x^2 - 200$

$$\begin{array}{c} 2(x^2 - 100) \\ \hline 2(x+10)(x-10) \end{array}$$

d) $-x^2 + 16$

$$\begin{array}{c} -(\sqrt{x^2 - 16}) \\ \hline -(x+4)(x-4) \end{array}$$

We can use this strategy to factor, even when the terms are not perfect squares. This just means that we will have radicals in our factors. What do we look for first when we factor? GCF

2) Factor each expression.

a) $\sqrt{x^2 - 21} < 3$

$$\boxed{(x+\sqrt{21})(x-\sqrt{21})}$$

b) $2x^2 - 24$

$$\begin{array}{c} 2(x^2 - 12) \\ \hline 2(x+2\sqrt{3})(x-2\sqrt{3}) \end{array}$$

$$\begin{array}{c} \sqrt{12} \\ 4\sqrt{3} \\ (22) \end{array}$$

* Be sure to simplify square roots

c) $5x^2 - 10$

$$\begin{array}{c} 5(x^2 - 2) \\ \hline 5(x+\sqrt{2})(x-\sqrt{2}) \end{array}$$

d) $\sqrt{x^2 - 27} < 9 < \frac{3}{3}$

$$\boxed{(x+3\sqrt{3})(x-3\sqrt{3})}$$

Using **difference** of squares to factor means that there must be subtraction in order to use it. If there is addition between your terms, then we can rewrite it as a difference.

Example: $x^2 + 16 = \sqrt{x^2} - \sqrt{(-16)} = \boxed{(x+4i)(x-4i)}$

Sum of squares

$(a^2 + b^2) = (a+bi)(a-bi)$

So, if there is addition between your terms, what kind of factors will you have? Imaginary

3) Factor each expression. What do you look for first when you factor? GCF

a) $x^2 + 25$

$\sqrt{x^2} - \sqrt{(-25)}$

$\boxed{(x+5i)(x-5i)}$

b) $16x^2 + 49$

$\sqrt{16x^2} - \sqrt{(-49)}$

$\boxed{(4x+7i)(4x-7i)}$

c) $-x^2 - 36$

$-(x^2 + 36)$

$-(\sqrt{x^2} - \sqrt{(-36)})$

$\boxed{-(x+6i)(x-6i)}$

d) $9x^3 + 54x$

$9x(x^2 + 6)$

$9x(\sqrt{x^2} - \sqrt{(-6)})$

$\boxed{9x(x+i\sqrt{6})(x-i\sqrt{6})}$

e) $4x^2 + 40$

$4(x^2 + 10)$

$4(\sqrt{x^2} - \sqrt{(-10)})$

$\boxed{4(x+i\sqrt{10})(x-i\sqrt{10})}$

f) $-2x^2 - 36$

$-2(x^2 + 18)$

$-2(\sqrt{x^2} - \sqrt{(-18)})$

$\boxed{-2(x+3i\sqrt{2})(x-3i\sqrt{2})}$

$\sqrt{18}$
 $\hat{9}^2$
 $\underbrace{3 \ 3}$

$= 3\sqrt{2}$