3.1 Greatest Common Factor

1) Simplify.

a. $2(5x^2 - 3x + 7)$

$10x^2 - 6x + 14$

b. $x(x - 4)$

$x^2 - 4x$

c. $3x^2(x^2 - 2x + 8)$

$3x^4 - 6x^3 + 24x^2$

2) Find the term that makes the two expressions equivalent.

d. $x^2 + \underline{3x} - 40 = (x + 8)(x - 5)$

$x^2 - 5x + 8x - 40$

e. $x^2 \underline{-7x} - 30 = (x - 10)(x + 3)$

$x^2 + 3x - 10x - 30$

We are used to starting with two multiples and distributing, but sometimes we want to work backwards from an expression to see the multiples that it came from.

Factoring: Undoing distribution, breaking an expression down to its multiples

The reason we care so much about factoring is because it helps us find solutions to equations. We will talk about that more in unit 4.

There are lots of different strategies to factor, but the first method you should **always** check for is the **greatest common factor.**

Greatest Common Factor: The largest multiple each term has in common

Standard Form

$ax^2 + bx + c$

Things to look for with GCFs:

1) Coefficients
2) Negatives
3) Variables

Coefficients

To check if there is a coefficient as a GCF, look for the biggest number that every coefficient is divisible by.

3) Factor the greatest common factor out of each expression.

a. $10x^2 - 6x + 14$ GCF: 2

$\boxed{2(5x^2 - 3x + 7)}$

Put 2 out front of parentheses & divide each term by 2

b. $15x^3 - 5x^2 - 20x + 5$ GCF: 5

$\boxed{5(3x^3 - x^2 - 4x + 1)}$

c. $27x + 30$ GCF: 3

$\boxed{3(9x + 10)}$

d. $6x^2 + 18x - 24$ GCF: 6

$\boxed{6(x^2 + 3x - 4)}$

Negatives

If the first term of an expression is negative, then we will factor out the negative. Because of this, it is *super important* that your expression is in standard form before you start factoring.

Highest exponent to lowest exponent

4) Factor the greatest common factor out of each expression.

a. $-x^2 - 4x + 6$

$\boxed{-(x^2+4x-6)}$

b. $3b^2 - 2b^4 - 7b$

$-2b^4 + 3b^2 - 7b$

$\boxed{-b(2b^3-3b+7)}$

c. $8m^3 - 2m^8 + 7m^6$

$-2m^8 + 7m^6 + 8m^3$

$\boxed{-m^3(2m^5-7m^3-8)}$

d. $-5y^6 + y^3 - y$

$\boxed{-y(5y^5+y^2-1)}$

Variables

If every term has the same variable (even if the exponents are different), then you will have a variable as a GCF. Whatever the smallest exponent is is the amount of the variable that you can factor out from each term.

5) Factor the greatest common factor out of each expression

a. $8m^8 - m^6$

$m^6(8m^2-1)$

b. $6x^5 + 2x^4 - x^3$

$x^3(6x^2+2x-1)$

c. $-3x^3 + 7x^4 + 24x^2$

$7x^4 - 3x^3 + 24x^2$

$\boxed{x^2(7x^2-3x+24)}$

d. $-7y^4 + 21y^7 + 2y^5$ * Standard form

$21y^7 + 2y^5 - 7y^4$

$\boxed{y^4(21y^3+2y-7)}$

Now that we've seen each of these individually, let's start putting them all together:

6) Factor the greatest common factor out of each expression.

a. $8a^8 - 4a^4 + 16a^2$

$\boxed{4a^2(2a^6-a^2+4)}$

b. $-x^5 + 7x^4 - 2x$

$\boxed{-x(x^4-7x^3+2)}$

c. $25c^4 + 15c^6 + 40c^5$

$15c^6 + 40c^5 + 25c^4$

$\boxed{5c^4(3c^2+8c+5)}$

d. $3m^3 - 12m^6$

$-12m^6 + 3m^3$

$\boxed{-3m^3(4m^3-1)}$

e. $36c^5 + 60c^7 - 12c^9$

$-12c^9 + 60c^7 + 36c^5$

$\boxed{-12c^5(c^4-5c^2-3)}$

f. $x^2 - 7x$

$\boxed{x(x-7)}$

* Once you have the GCF, divide every term by the GCF

Factoring by Grouping

If you are given four terms and are asked to factor, you may want to consider factoring by grouping. When we factor by grouping, we are going to split the expression into groups and take the GCF out of each group.

What is the first thing you always look for when you factor? GCF

Steps to factoring by grouping:

1) Look for a GCF & factor it out if needed
2) Split expression in half & factor out the GCF from each half
3) Factor out the shared binomial

* if the binomial is not the same, go back & check your work

a) $4x^2 + 20x - 3xy - 15y$ No GCF

$4x(x+5) - 3y(x+5)$

$\boxed{(x+5)(4x-3y)}$

The stuff outside the parentheses combines into its own parentheses

b) $3x^3 - 6x^2 + 15x - 30$ GCF: 3

$3(x^3 - 2x^2 + 5x - 10)$

$3(x^2(x-2) + 5(x-2))$

$\boxed{3(x-2)(x^2+5)}$

The GCF stays out front for the entirety of the problem

c) $4x^3 + 8x^2 + x + 2$ No GCF

$4x^2(x+2) + 1(x+2)$

$\boxed{(x+2)(4x^2+1)}$

d) $3xy - 21y + 5x - 35$ No GCF

$3y(x-7) + 5(x-7)$

$\boxed{(x-7)(3y+5)}$

e) $x^2y - 3x^2 - 8y + 24$ No GCF

$x^2(y-3) - 8(y-3)$

$\boxed{(x^2-8)(y-3)}$

f) $8x^3 + 6x^2 - 24x - 18$ GCF: 2

$2(4x^3 + 3x^2 - 12x - 9)$

$2(x^2(4x+3) - 3(4x+3))$

$\boxed{2(x^2-3)(4x+3)}$