

Unit 5 Functions Review

Transform the given function $f(x)$ as described and write the resulting function as an equation.

1) $f(x) = |x|$

expand vertically by a factor of 3
reflect across the x-axis
translate right 2 units
translate up 3 units

$$g(x) = -3|x - 2| + 3$$

2) $f(x) = x^2$

compress vertically by a factor of 2
reflect across the x-axis
translate left 2 units
translate up 2 units

$$g(x) = -\frac{1}{2}(x + 2)^2 + 2$$

3) $f(x) = x^3$

compress vertically by a factor of 2
reflect across the x-axis
translate left 3 units
translate down 1 unit

$$g(x) = -\frac{1}{2}(x + 3)^3 - 1$$

Describe the transformations necessary to transform the graph of $f(x)$ into that of $g(x)$.

4) $f(x) = \sqrt{x}$
 $g(x) = -2\sqrt{x + 1} + 2$

stretch vertically by a factor of 2
reflect across the x-axis
translate left 1 unit
translate up 2 units

5) $f(x) = |x|$
 $g(x) = -\frac{1}{2} \cdot |x + 2| - 3$

compress vertically by a factor of 2
reflect across the x-axis
translate left 2 units
translate down 3 units

6) $f(x) = x^2$

$$g(x) = -\frac{1}{3}(x + 1)^2 - 3$$

compress vertically by a factor of 3
reflect across the x-axis
translate left 1 unit
translate down 3 units

For each problem, find the average rate of change of the function over the given interval.

7) $f(x) = -x^2 + 1$; $[-2, 1]$

1

8) $f(x) = 2x^2 + 2x + 2$; $[-2, -\frac{5}{3}]$

$-\frac{16}{3}$

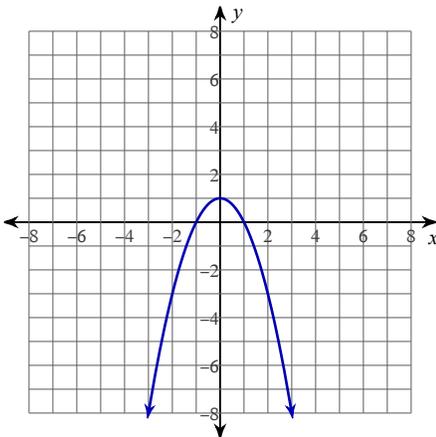
9) $y = -2|x - 3| - 1$; $[-1, 4]$

$\frac{6}{5}$

10) $y = \frac{1}{2} \cdot 4^x$; $[2, 4]$

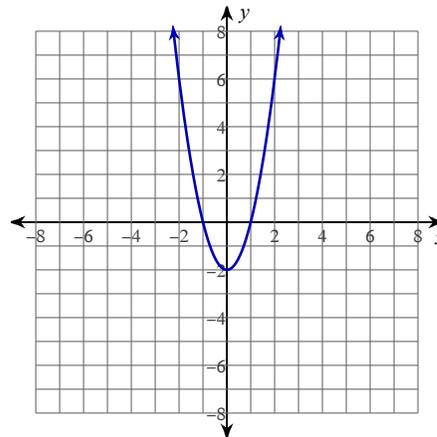
60

11) $y = -x^2 + 1$; $[-1, 0]$



1

12) $y = 2x^2 - 2$; $[0, 2]$



4

13) A savings account starts with \$1,500. It gains 2.5% interest each month. How much is in the account after one year?

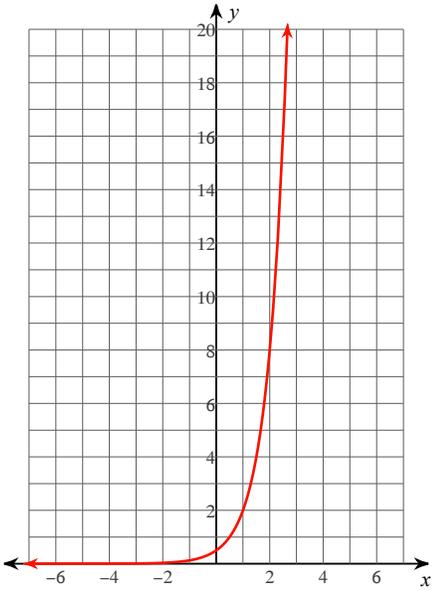
\$2,017.33

14) A radioactive substance decays by 10% each year. If there was originally 50 grams of the substance, how much would be left after 5 years?

29.52 grams

Sketch the graph of each function. Then state the domain, range, intervals of increasing and decreasing, and the end behavior.

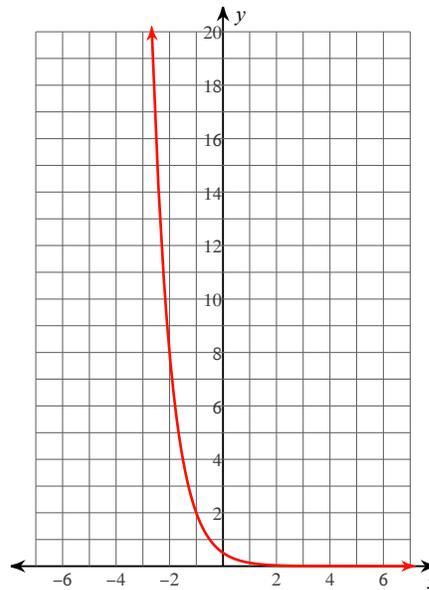
15) $y = \frac{1}{2} \cdot 4^x$



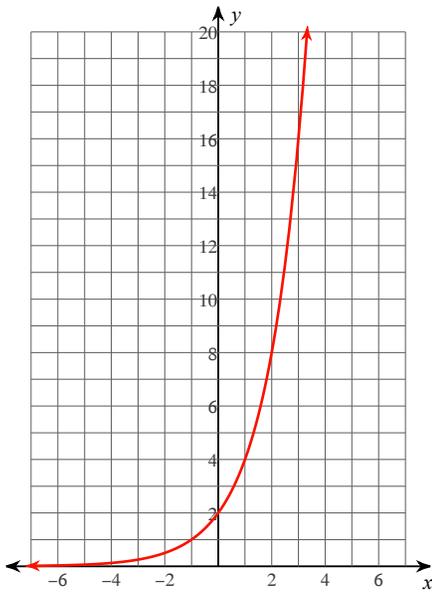
- 16) a. Domain: a. $(-\infty, \infty)$
 b. Range: b. $(0, \infty)$
 c. Increasing: c. $(-\infty, \infty)$
 d. Decreasing: d. None
 e. End behavior: e. As $x \rightarrow -\infty, y \rightarrow 0$
 As $x \rightarrow \infty, y \rightarrow \infty$

- 17) a. Domain: a. $(-\infty, \infty)$
 b. Range: b. $(0, \infty)$
 c. Increasing: c. None
 d. Decreasing: d. $(-\infty, \infty)$
 e. End behavior: e. As $x \rightarrow -\infty, y \rightarrow \infty$
 As $x \rightarrow \infty, y \rightarrow 0$

18) $y = \frac{1}{2} \cdot \left(\frac{1}{4}\right)^x$



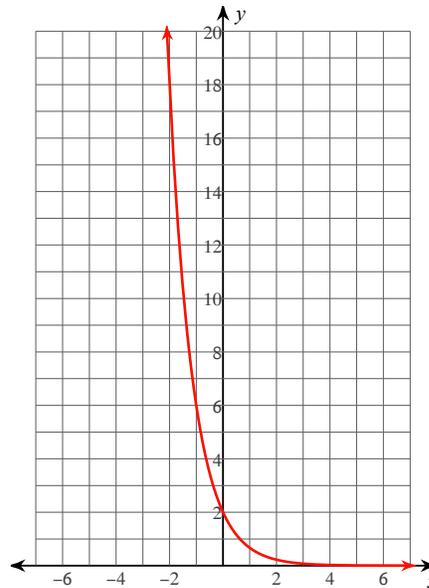
19) $y = 2 \cdot 2^x$



- 20) a. Domain: a. $(-\infty, \infty)$
 b. Range: b. $(0, \infty)$
 c. Increasing: c. $(-\infty, \infty)$
 d. Decreasing: d. None
 e. End behavior: e. As $x \rightarrow -\infty, y \rightarrow 0$
 As $x \rightarrow \infty, y \rightarrow \infty$

- 21) a. Domain: a. $(-\infty, \infty)$
 b. Range: b. $(0, \infty)$
 c. Increasing: c. None
 d. Decreasing: d. $(-\infty, \infty)$
 e. End behavior: e. As $x \rightarrow -\infty, y \rightarrow \infty$
 As $x \rightarrow \infty, y \rightarrow 0$

22) $y = 2 \cdot \left(\frac{1}{3}\right)^x$



Identify the initial value, growth or decay factor, and growth or decay rate.

- 23) $y = 12 \cdot 0.87^x$
 Initial: 12
 Decay factor: 0.87
 Decay rate: 13%

- 24) $y = 25 \cdot 0.93^{4x}$ Initial: 25
 Decay factor: 0.75
 Decay rate: 25%

- 25) $y = 1.07^{3x}$
 Initial: 1
 Decay factor: 1.23
 Decay rate: 23%

- 26) $y = 35 \cdot 1.14^x$ Initial: 35
 Decay factor: 1.14
 Decay rate: 14%

Evaluate.

$$27) h(x) = \begin{cases} 2|x| - 3, & x \leq -4 \\ 3x^2 - 1, & x = -3 \\ -x^2 + 3, & x \geq 0 \end{cases}$$

a. $h(-2)$

b. $h(-3)$

c. $h(2)$

d. $h(-10)$

a. Undefined

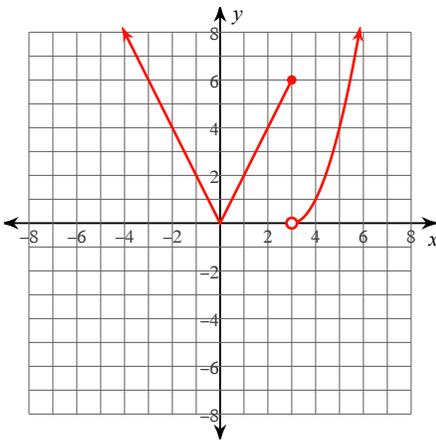
b. 26

c. -1

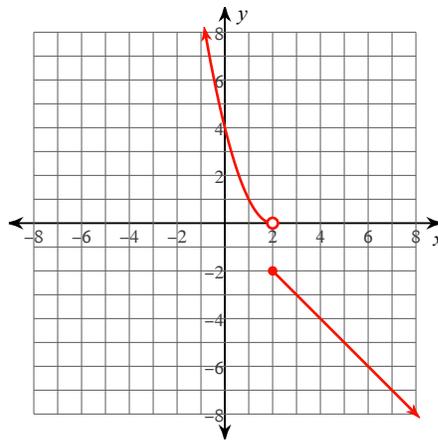
d. 17

Sketch the graph of each function.

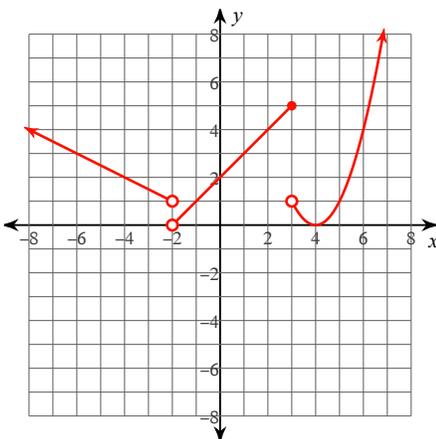
$$28) g(x) = \begin{cases} 2|x|, & x \leq 3 \\ (x-3)^2, & x > 3 \end{cases}$$



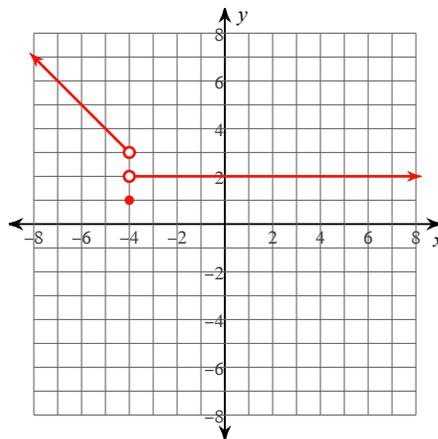
$$29) g(x) = \begin{cases} (x-2)^2, & x < 2 \\ -|x|, & x \geq 2 \end{cases}$$



$$30) g(x) = \begin{cases} \frac{|x|}{2}, & x < -2 \\ x+2, & -2 < x \leq 3 \\ (x-4)^2, & x > 3 \end{cases}$$

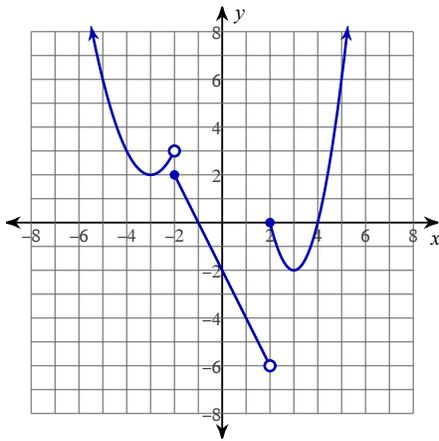


$$31) w(x) = \begin{cases} -x-1, & x < -4 \\ (x+5)^2, & x = -4 \\ 2, & x > -4 \end{cases}$$



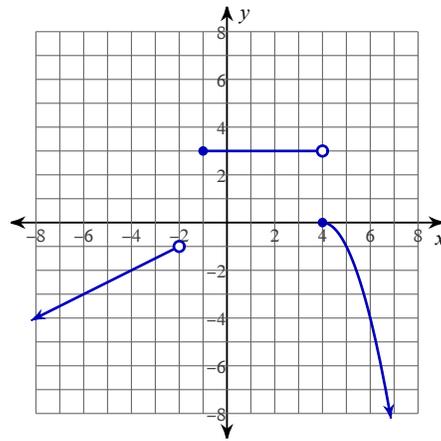
Write an equation for each piecewise function.

32)



$$f(x) = \begin{cases} (x+3)^2 + 2, & x < -2 \\ -2x - 2, & -2 \leq x < 2 \\ 2(x-3)^2 - 2, & x \geq 2 \end{cases}$$

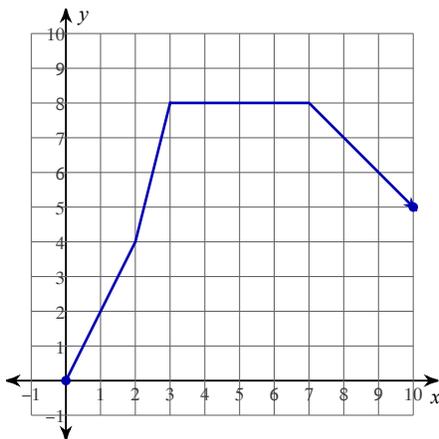
33)



$$g(x) = \begin{cases} \frac{1}{2}x, & x < -2 \\ 3, & -1 \leq x < 4 \\ -(x-4)^2, & x \geq 4 \end{cases}$$

Write a scenario that can be modeled by the graph.

34)



Jane is warming up for a run. She jogs at a certain pace for the first 2 minutes, increases her speed for the next minute, and then she rests and stretches for 5. She then starts jogging back to where she started at a cool-down pace.

Determine if each situation represents linear, quadratic, exponential, or none.

35) A salary scale starts at \$45,000 plus 10% commission for every sale.

Linear

36) $\{(-1, 4), (0, 8), (2, 32), (4, 128)\}$

Exponential

37) $\{(-4, -2), (-2, 8), (-1, -3), (0, 12)\}$

None

38) $\{(-2, 16), (-1, 7), (0, 0), (1, -5), (2, -8)\}$

Quadratic

39) A radioactive substance decays by 50% every 12 years.

Exponential