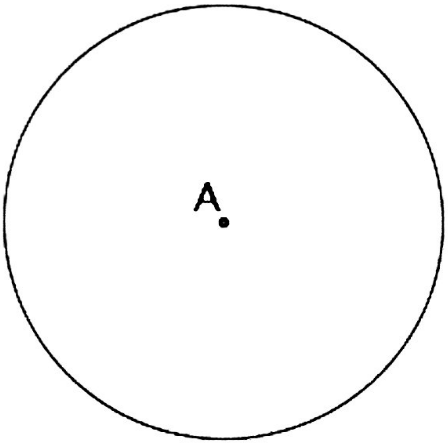
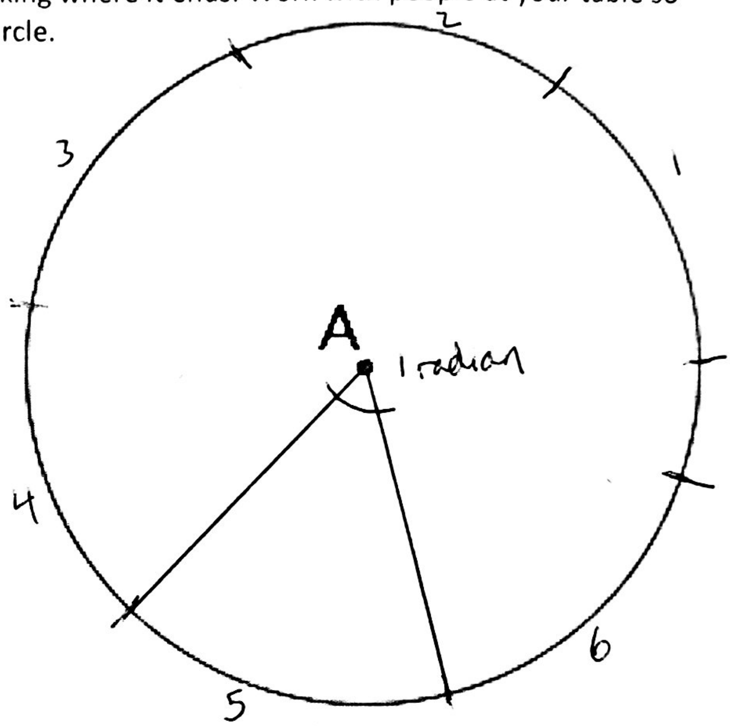
8.5 Radians

What is a radian? To find out, cut a pipe cleaner so you have a piece that is the size of the radius. Take that piece and wrap it around the edge of your circle, marking where it ends. Work with people at your table so there is at least one person working on each size of circle.





1) How many radius lengths did it take to get around the circle? A little more than 6

2) Recall that the circumference of a circle is $2\pi r$, where r is the radius of the circle. Where do you think this formula came from? (Hint: What is the value of 2π?)

$2\pi = 6.28$ $6.28r$

It takes 2π radius lengths to get around the full circle

$$\boxed{\pi = \frac{\text{circumference}}{\text{diameter}}}$$

3) Draw two segments that connect the ends of a radius length around the edge of a circle to the center of the circle. The angle that you have just created is a **radian.** Describe what a radian is in your own words. An angle that opens as wide as the radius.

4) How many radians does it take to make a full rotation? How many degrees? $2\pi / 360°$

5) Write a ratio that compares radians to degrees. Be sure to simplify the ratio. $\frac{2\pi}{360} = \frac{\pi}{180}$

6) How many radians is 60°? How about 270°?

$\frac{60° \mid \pi \text{ rad}}{\mid 180°} = \frac{60\pi}{180} = \boxed{\frac{\pi}{3} \text{ rad}}$ $270 \cdot \frac{\pi}{180} = \boxed{\frac{3\pi}{2} \text{ rad}}$

7) How many degrees is $\frac{\pi}{2}$ radians? How about $\frac{11\pi}{6}$ radians?

$\frac{\frac{\cancel{\pi}}{2} \text{ rad} \mid 180°}{\mid \cancel{\pi} \text{ rad}} = \frac{180}{2} = \boxed{90°}$ $\frac{11\cancel{\pi}}{6} \cdot \frac{180}{\cancel{\pi}} = \boxed{330°}$

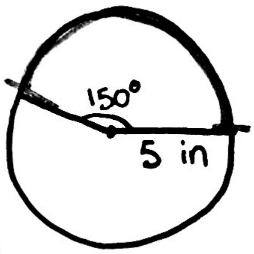
Angle Conversions

Radians → Degrees	Degrees → Radians
Multiply by $\frac{180°}{\pi}$	Multiply by $\frac{\pi}{180°}$

Arc Length - portion of the circumference

set up an equation that represents a fraction of the circumference

ex:



*Type in all except π in calculator

Fraction of Circumference

$$\frac{150}{360} \cdot 2\pi(5)$$

*Exact: $\boxed{\frac{25\pi}{6} \text{ in}}$
(not rounded)

Give both →

Approx: $\boxed{13.09 \text{ in}}$
(rounded)

Type everything in calculator

*** Arc Length**

Degrees: $\frac{\theta}{360} \cdot 2\pi r$

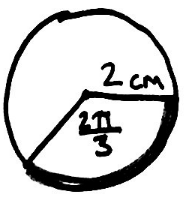
Radians: θr

If we do this same format with radians:

Fraction of Circumference

$$\frac{\theta}{\cancel{2\pi}} \cdot \cancel{2\pi} r$$
$$= \theta r$$

ex:



θr

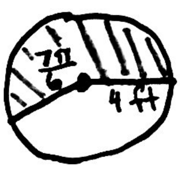
$\frac{2\pi}{3} \cdot 2$

Exact: $\frac{4\pi}{3}$ in

Approx: 4.19 in

Sector Area - portion of total area

ex:



Fraction of Area

$$\frac{\theta}{2\cancel{\pi}} \cdot \cancel{\pi} r^2 = \frac{\theta r^2}{2}$$

$$\frac{\frac{7\pi}{6}(4)^2}{2}$$

Exact: $\frac{28\pi}{3}$ ft^2

Approx: 29.32 ft^2

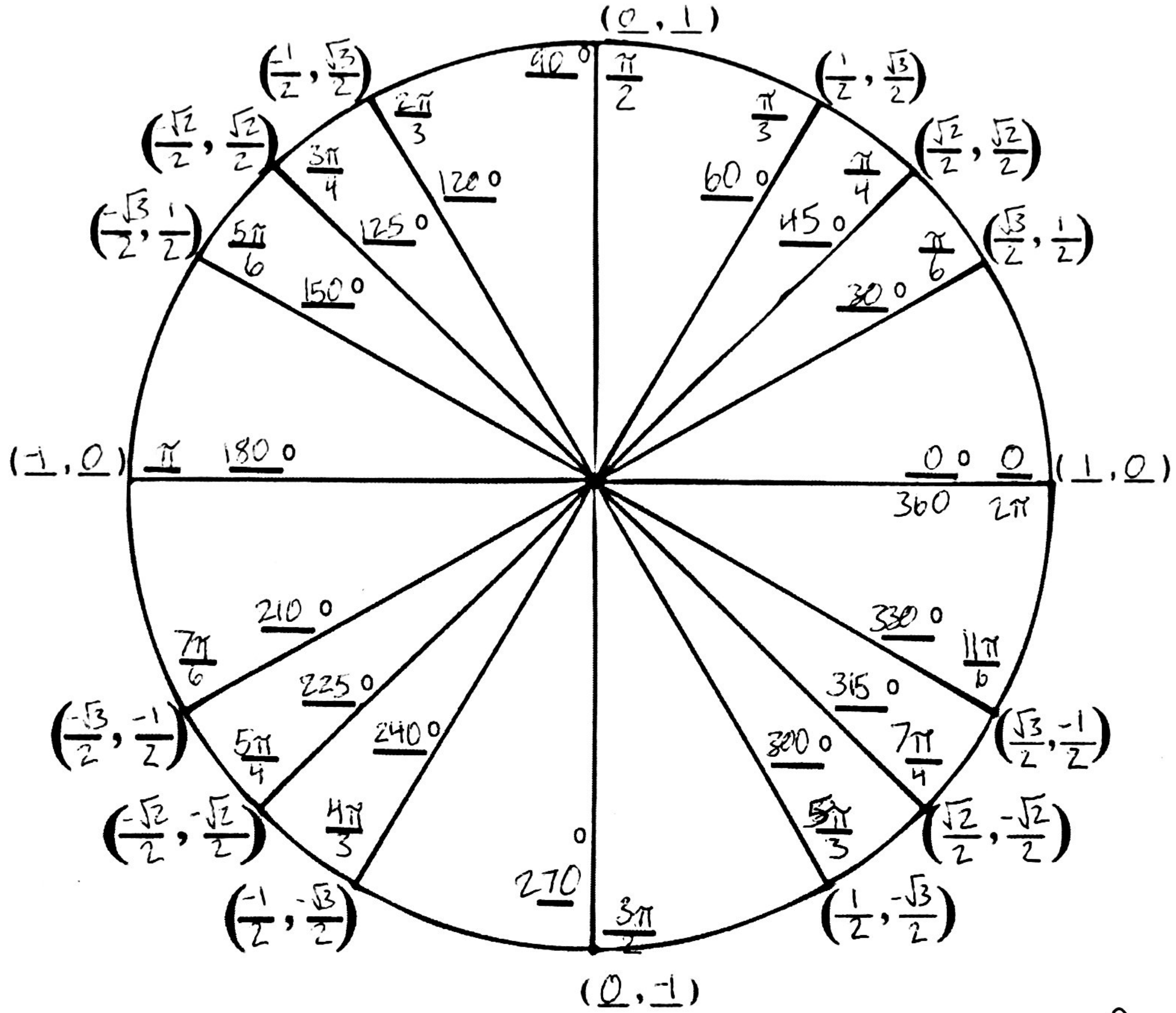
*** Sector Area**

Degrees: $\frac{\theta}{360} \cdot \pi r^2$

Radians: $\frac{\theta r^2}{2}$

*If you can switch between radians and degrees, you only need to memorize one formula

Unit Circle, Fill in the blank



$\sin\theta = y$
$\cos\theta = x$
$\tan\theta = \frac{y}{x}$

Find the exact value of each trig function.

1) $\sin\frac{\pi}{4}$ $\boxed{\frac{\sqrt{2}}{2}}$

y of $\frac{\pi}{4}$

2) $\cos\frac{5\pi}{6}$ $\boxed{-\frac{\sqrt{3}}{2}}$

x of $\frac{5\pi}{6}$

3) $\tan\frac{3\pi}{2}$ $\boxed{\text{Undefined}}$

4) $\sin\frac{11\pi}{6}$ $\boxed{-\frac{1}{2}}$

5) $\cos\frac{7\pi}{6}$ $\boxed{-\frac{\sqrt{3}}{2}}$

6) $\tan\frac{5\pi}{4}$ $\boxed{1}$