

8.4 Exponential Functions

Simplify the following:

$2^0 =$

$2^{-1} =$

$4^{-2} =$

$2^4 \cdot 2^6 =$

$(6^5)^3 =$

$(3^2)^4 =$

When the exponent is a variable, it is called an exponential function.

a is the initial value (y-intercept)

$$f(x) = a \cdot b^x$$

b is the growth/decay factor

base (multiplier)

To determine what makes our b value a growth factor or a decay factor, find each of the following

$$f(x) = 2^x \quad a = 1 \quad b = 2$$

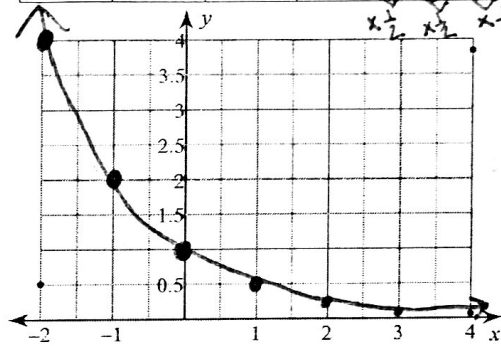
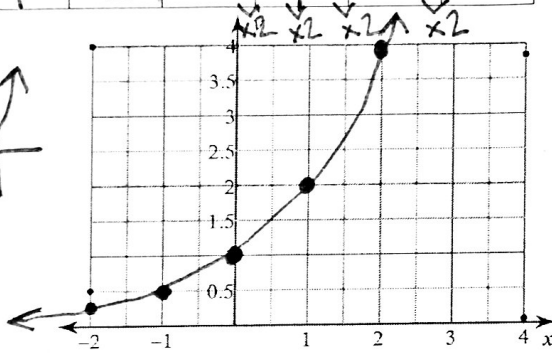
$$= 1 \cdot 2^x$$

$$g(x) = (1/2)^x \quad a = 1 \quad b = 1/2$$

$$= 1 \cdot (1/2)^x$$

x	-2	-1	0	1	2	3	4
y	1/4	1/2	1	2	4	8	16

x	-2	-1	0	1	2	3	4
y	4	2	1	1/2	1/4	1/8	1/16



Growth Factor

Decay factor

$b > 1$

$b < 1$

Let's see if you can figure out how to find the growth/decay rate by looking at the following examples:

$f(x) = 3 \cdot 1.23^x$
 Growth factor: 1.23
 Growth rate: 23%

$g(x) = 0.5 \cdot 1.64^x$
 GF: 1.64
 Growth rate: 64%

$h(x) = 2 \left(\frac{3}{4}\right)^x$

Decay rate: 25%

$k(x) = 0.4 \cdot 0.39^x$

Decay rate: 61%

$1.23 - 1 = .23$

$1.64 - 1 = .64$

$DF: \frac{3}{4} = .75$

$DF: 0.39$

$1 - .75 = .25$

$1 - .39 = .61$

Growth Rate

Decay rate

$b - 1$

$1 - b$

The growth/decay **RATE** is the percent change between each output value of the function.

Ex: Identify the initial value, the growth or decay factor, and the growth or decay rate for each of the functions below.

$$1) f(x) = 4 \cdot 0.78^x$$

Initial: 4

Decay factor: 0.78

Decay rate: 22%

$$1 - 0.78 = 0.22$$

$$-4) y = 1.5 \cdot 0.36^x$$

Initial: 1.5

DF: 0.36

DR: 64%

$$1 - 0.36 = 0.64$$

$$-2) y = 5 \cdot 1.47^x$$

Initial: 5

GF: 1.47

GR: 47%

$$3) g(t) = 0.6 \cdot 1.19^t$$

$$-5) h(x) = 3 \left(\frac{2}{5} \right)^x$$

Initial: 3

DF: $\frac{2}{5} = 0.4$

DR: 60%

$$1 - 0.4 = 0.6$$

$$-6) k(x) = 2 \cdot 2^x$$

Initial: 2

GF: 2

GR: 100%

$$2 - 1 = 1$$

Ex: Find the growth or decay rate factor for the functions below and state the growth or decay rate.

$$7) f(x) = (1.05^4)^x = 1.22^x$$

GF: 1.22

GR: 22%

$$-8) h(t) = (0.68^3)^t = 0.31^t$$

DF: 0.31

DR: 69%

$$1 - 0.31 = 0.69$$

$$-9) y = (1.46^3)^x = 3.11^x$$

GF: 3.11

GR: 211%

$$3.11 - 1 = 2.11$$

$$10) g(x) = 1.25^{2x}$$


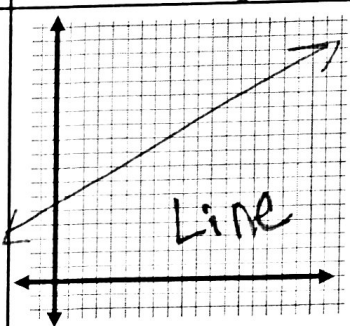
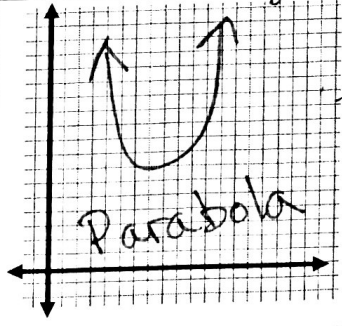
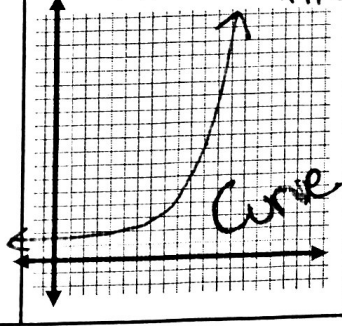
$$11) y = 0.46^{3x}$$

$$12) f(t) = 0.52^{4t}$$

Separate x by putting parentheses around everything else. Then type what is in the parentheses into your calculator.

Comparing Functions

Given below are 3 different situations, each one given in 4 different representations. Go through the table below decide how to tell which function family a situation would belong to for each representation. Make note of your findings in the table on the next page.

	<i>Slowest growth rate</i> Linear Situation	<i>2nd fastest growth rate</i> Quadratic Situation	<i>Fastest growth rate</i> Exponential Situation																																						
Scenario	<p><i>Constant</i></p> <p>John works at Blockbuster and receives a flat rate of \$20 each week along with \$10 for every <u>10x</u> extra hour he works overtime. Model his weekly earnings as a function of hours worked.</p>	<p>Blockbuster started out as a small company with few profits. They became one of the leading companies in movie rental. However, since Redbox... and the internet, their profits have significantly decreased. Most are now obsolete.</p> <p><i>Increases & decreases</i></p> 	<p>John's manager sees he has performed so well that he decides to change his pay scale. He will give John \$1 on the first day and <u>double the amount</u> each day afterward.</p> <p><i>Multiply/divide</i></p>																																						
Equation	$y = 10x + 20$ <p><i>Just x</i></p>	$y = -x^2 + 5x + 1$ <p><i>x²</i></p>	$y = 2^x$ <p><i>x is exponent</i></p>																																						
Table	<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr><th>Hour</th><th>Pay</th></tr> </thead> <tbody> <tr><td>0</td><td>20</td></tr> <tr><td>1</td><td>30</td></tr> <tr><td>2</td><td>40</td></tr> <tr><td>3</td><td>50</td></tr> <tr><td>4</td><td></td></tr> </tbody> </table> <p><i>Constant rate of change</i></p>	Hour	Pay	0	20	1	30	2	40	3	50	4		<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr><th>Year</th><th>Profits</th></tr> </thead> <tbody> <tr><td>-1</td><td>-17</td></tr> <tr><td>0</td><td>-7</td></tr> <tr><td>1</td><td>-1</td></tr> <tr><td>2</td><td>1</td></tr> <tr><td>3</td><td>-1</td></tr> <tr><td>4</td><td>-7</td></tr> </tbody> </table> <p><i>Vertex, increase/decrease 1-3-5 pattern 2nd rate of change is constant</i></p>	Year	Profits	-1	-17	0	-7	1	-1	2	1	3	-1	4	-7	<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr><th>Days</th><th>Pay</th></tr> </thead> <tbody> <tr><td>0</td><td>$\frac{1}{12}$</td></tr> <tr><td>1</td><td>\$1</td></tr> <tr><td>2</td><td>\$12</td></tr> <tr><td>3</td><td>\$144</td></tr> <tr><td>4</td><td></td></tr> </tbody> </table> <p><i>Multiply/divide each time</i></p>	Days	Pay	0	$\frac{1}{12}$	1	\$1	2	\$12	3	\$144	4	
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