8.3 Piecewise Functions

Up until now a function has been represented by a single equation. In many real-life problems, however, functions are represented by a combination of equations, each corresponding to a part of the domain. Such functions are called **PIECEWISE FUNCTIONS.** For example, the piecewise function given by

Domain

$$f(x) = \begin{cases} 2x - 1, & if\ x \le 1 \\ 3x + 1, & if\ x > 1 \end{cases}$$

Equations — where they happen

Is defined by two equations. One equation gives the values of $f(x)$ when x is less than or equal to 1, and the other equation gives the values of $f(x)$ when x is greater than 1.

*To evaluate an equation at a certain x-value of a piecewise function, first determine which equation to plug x into. That means you have to determine which domain your x falls in and plug it into the corresponding equation.

1. a) For $f(x) = \begin{cases} ① x^2 - 1, & if\ x \le 2 \\ ② 4x + 1, & if\ x > 2 \end{cases}$, evaluate when

(a) $x = 0$
$0 \le 2$, so plug into ①
$(0)^2 - 1 = 0 - 1 = \boxed{-1}$

(b) $x = 2$
$2 \le 2$, so plug into ①
$(2)^2 - 1 = 4 - 1 = \boxed{3}$

(c) $x = 4$.
$4 > 2$, so plug into ②
$4(4) + 1 = 16 + 1 = \boxed{17}$

b) For $f(x) = \begin{cases} ① (x+1)^2, & -5 \le x < -1 \\ ② x - 1, & -1 < x \le 3 \\ ③ 2|x|, & x > 3 \end{cases}$, find

$\le x <$ means x is between...

(a) $f(-4)$
① $(-4+1)^2$
$(-3)^2 = \boxed{9}$

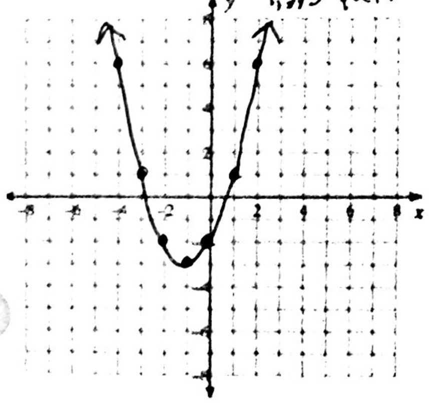
(b) $f(0)$
② $0 - 1 = \boxed{-1}$

(c) $f(3)$
② $3 - 1 = \boxed{2}$

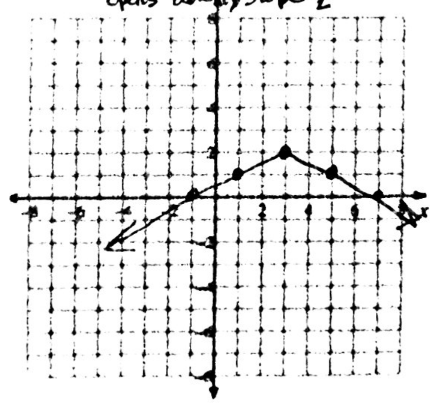
(d) $f(18)$
③ $2|18| = 2(18) = \boxed{36}$
Absolute value just makes things positive

Let's do a quick review of graphing transformations before we continue:

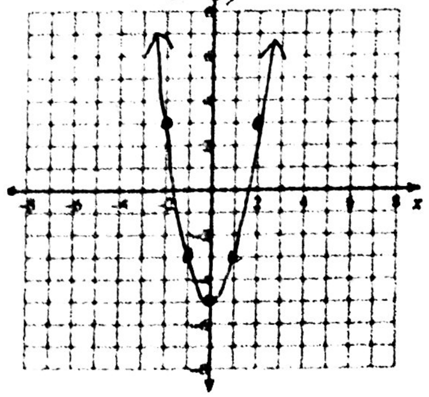
a. $y = (x+1)^2 - 3$
Left 1, down 3
1, 3, 5 pattern



b. $y = -\frac{1}{2}|x - 3| + 2$
Right 3, up 2
Opens down, slope $\frac{1}{2}$



c. $y = 2x^2 - 5$
Down 5
a = 2, 2, 6, 10 pattern



Graphing piecewise functions can be done in one of two ways:

1) Graph all equations and erase any pieces that lie outside of the given domain

   or

2) Draw lines where the graph changes and only draw the graphs within the boundaries

Open hole:

$<$, $>$

Closed point:

$\leq$, $\geq$

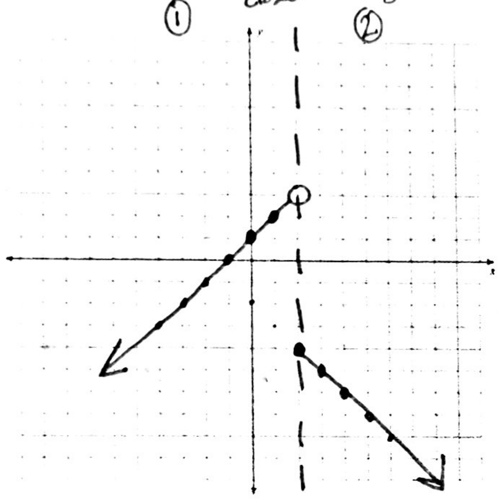
3. Graph the following piecewise functions.

a) $y = \begin{cases} x + 1, & x < 2 \\ -x - 2, & x \geq 2 \end{cases}$

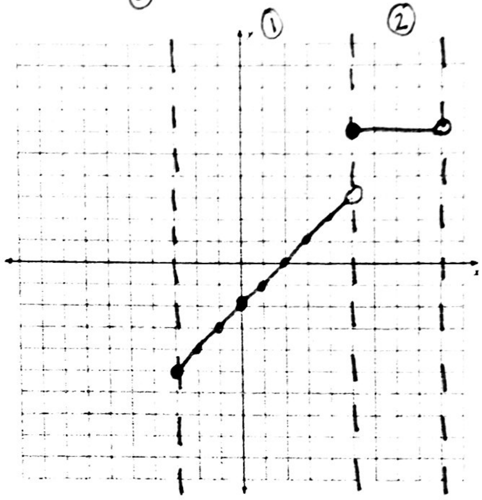
open

closed

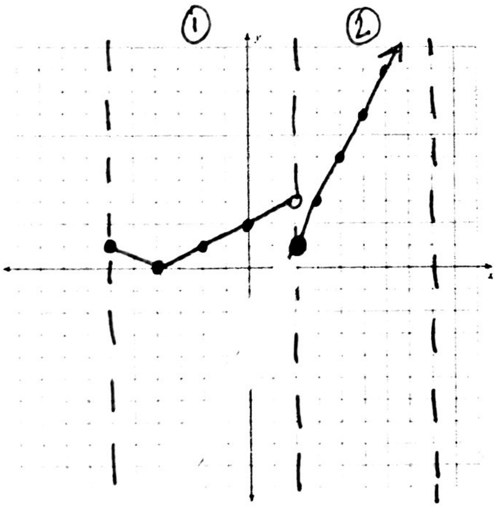
Draw vertical dotted lines from numbers given in domain



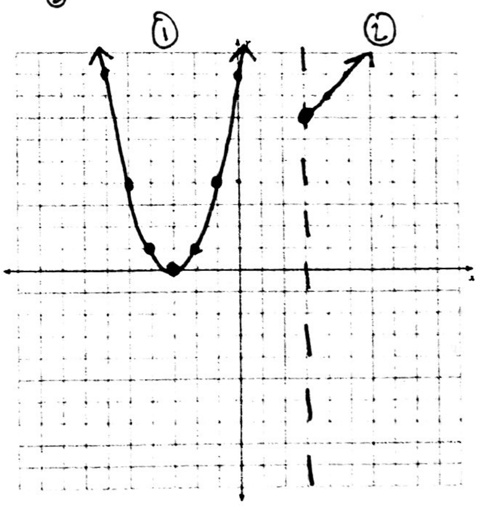
b) $y = \begin{cases} x - 2, & -3 \leq x < 5 \\ 6, & 5 \leq x < 9 \end{cases}$



c) $y = \begin{cases} \frac{1}{2}|x + 4|, & -6 \leq x < 2 \\ 2x - 3, & 2 \leq x < 8 \end{cases}$

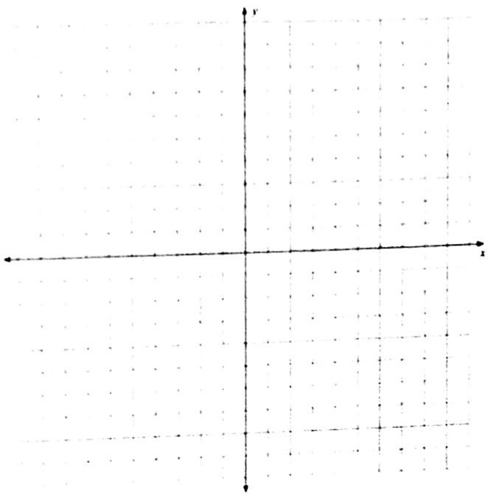


d) $f(x) = \begin{cases} (x + 3)^2, & x < 3 \\ |x| + 4, & x \geq 3 \end{cases}$

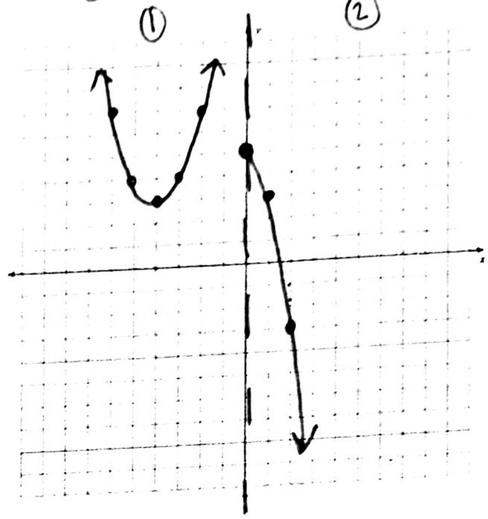


② Graph as you would normally, starting on the x-axis, and then erase what you don't need

d) $f(x) = \begin{cases} 2(x+4)^2, & x < -2 \\ 3|x| + 2, & x \geq -2 \end{cases}$

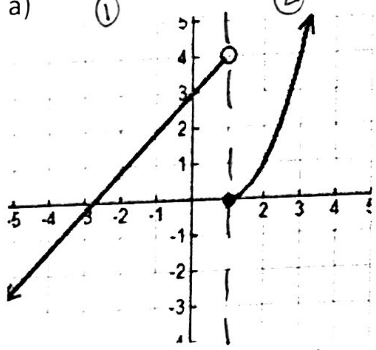


e) $f(x) = \begin{cases} ①\ (x+4)^2 + 3, & x < 0 \\ ②\ -2x^2 + 5, & x \geq 0 \end{cases}$



2. What equation would you use to describe the graphs below:

a)



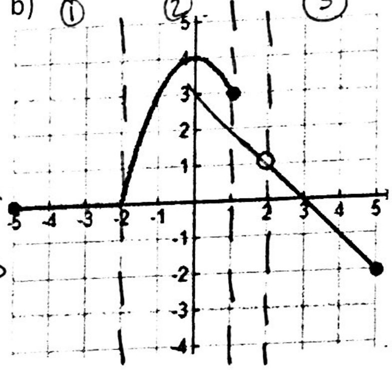
1) Draw dotted lines wherever graph suddenly changes shape
2) Write intervals
 • look at open/closed points
3) Write equations

$f(x) = \begin{cases} ①\ x+3, & x<1 \\ (x-1)^2, & x \geq 1 \end{cases}$

Shift right 1

$\geq$ since it has a closed point

b)



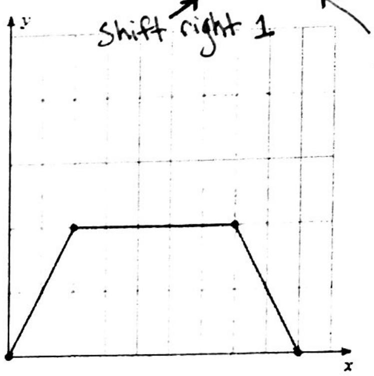
* If there is more than one dotted line, you'll probably use a combination of $x >$, $x <$, and $< x <$

$f(x) = \begin{cases} ①\ 0, & -5 \leq x < -2 \\ ②\ -x^2+4, & -2 \leq x < 1 \\ ③\ -x+3 & 2 < x \leq 5 \end{cases}$

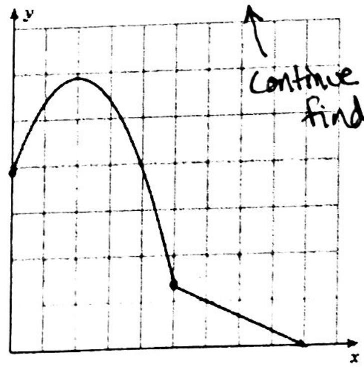
Continue slope to find y-int

If you can't tell which interval has the closed point, just choose one of them

c)



d)



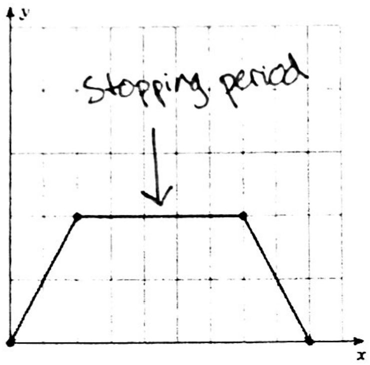
Example 4: Sketch a function that will model the following situations

a) John is taking his dog for a walk. He stops to tie his shoes, then continues walking.

b) Mandee is running down the beach towards the waters at a constant speed. She continues to run giving the same effort as before

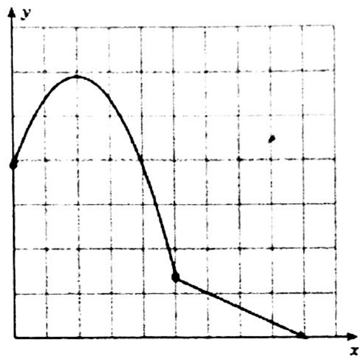
Example 5: Describe the situation that could be modeled by the following graph:

a)



Jane walks to the park. She stays to play for a while, then walks home.

b)



A ball is thrown from the top of a roof. It lands on the driveway and then rolls down the driveway.