

### 8.3 Piecewise Functions

Up until now a function has been represented by a single equation. In many real-life problems, however, functions are represented by a combination of equations, each corresponding to a part of the domain.

Such functions are called **PIECEWISE FUNCTIONS**. For example, the piecewise function given by

$$f(x) = \begin{cases} 2x - 1, & \text{if } x \leq 1 \\ 3x + 1, & \text{if } x > 1 \end{cases}$$

**Domain**

Is defined by two equations. One equation gives the values of  $f(x)$  when  $x$  is less than or equal to 1, and the other equation gives the values of  $f(x)$  when  $x$  is greater than 1.

\*To evaluate an equation at a certain  $x$ -value of a piecewise function, first determine which equation to plug  $x$  into. That means you have to determine which domain your  $x$  falls in and plug it into the corresponding equation.

1. a) For  $f(x) = \begin{cases} x^2 - 1, & \text{if } x \leq 2 \\ 4x + 1, & \text{if } x > 2 \end{cases}$ , evaluate when

(a)  $x = 0$   
 ①  $0 \leq 2$

$$(0)^2 - 1 = 0 - 1 = \boxed{-1}$$

(b)  $x = 2$   
 ①  $2 \leq 2$

$$(2)^2 - 1 = 4 - 1 = \boxed{3}$$

(c)  $x = 4$ .  
 ②  $4 > 2$

$$4(4) + 1 = 16 + 1 = \boxed{17}$$

b) For  $f(x) = \begin{cases} (x+1)^2, & -5 \leq x < -1 \\ x - 1, & -1 < x \leq 3 \\ 2|x|, & x > 3 \end{cases}$ , find

< $x$ < "x is between"

(a)  $f(3)$

$$\textcircled{2} \quad 3 - 1 = \boxed{2}$$

$$-1 < 3 \leq 3$$

(b)  $f(0)$

$$\textcircled{2} \quad 0 - 1 = \boxed{-1}$$

$$-1 < 0 \leq 3$$

(c)  $f(-4)$

$$\textcircled{1} \quad (-4+1)^2 = (-3)^2 \\ = \boxed{9}$$

$$-5 \leq -4 < -1$$

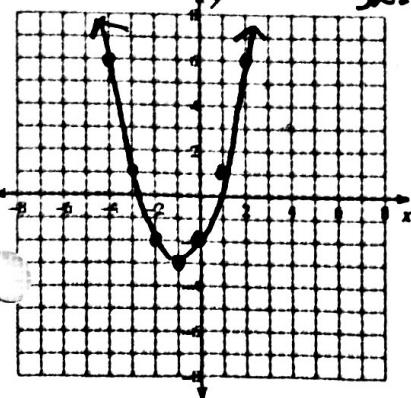
(d)  $f(18)$

$$\textcircled{3} \quad 2|18| = \boxed{36}$$

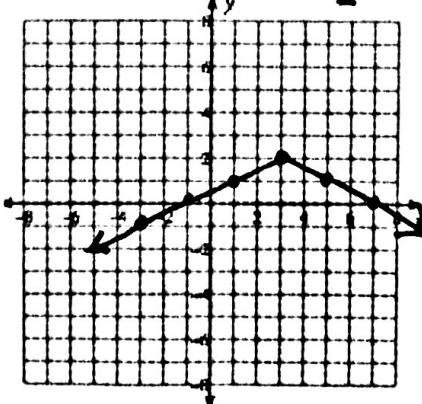
$$18 > 3$$

Let's do a quick review of graphing transformations before we continue:

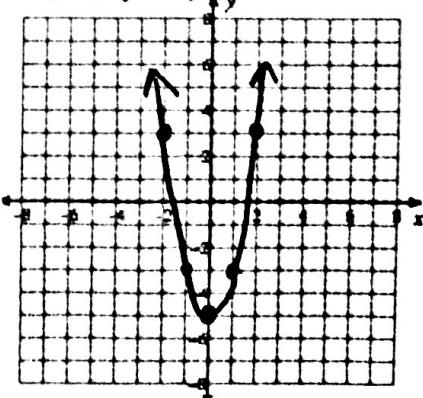
a.  $y = |(x+1)^2 - 3$   
 $\checkmark: (-1, -3), a=1 \quad \begin{matrix} 1a=1 \\ 3a=3 \\ 5a=5 \end{matrix}$



b.  $y = -\frac{1}{2}|x-3| + 2$   
 $\checkmark: (3, 2), a = -\frac{1}{2}$



c.  $y = 2x^2 - 5$   
 $\checkmark: (0, -5), a=2 \quad \begin{matrix} 1a=2 \\ 3a=6 \end{matrix}$

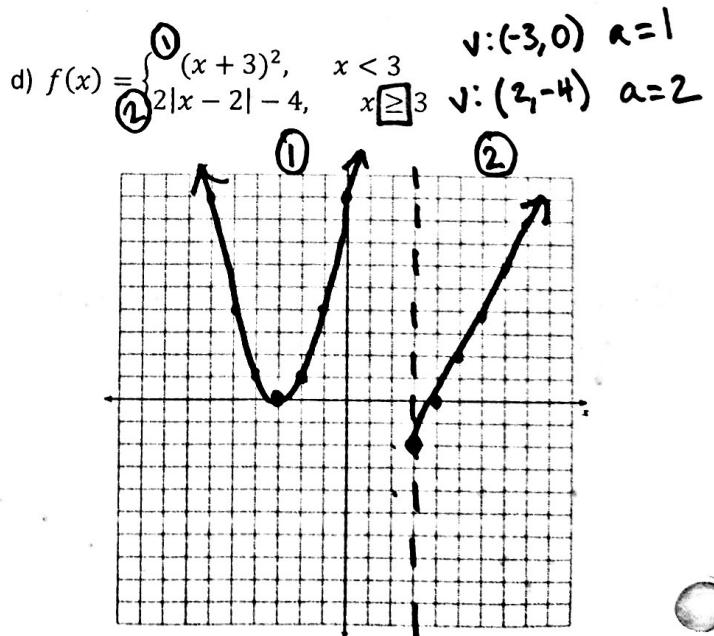
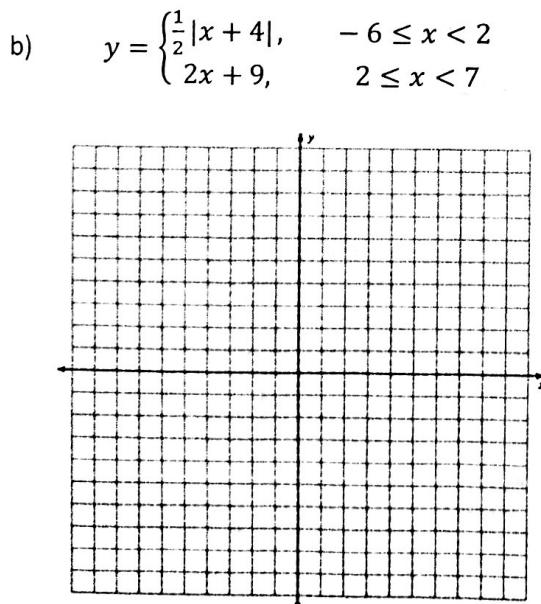
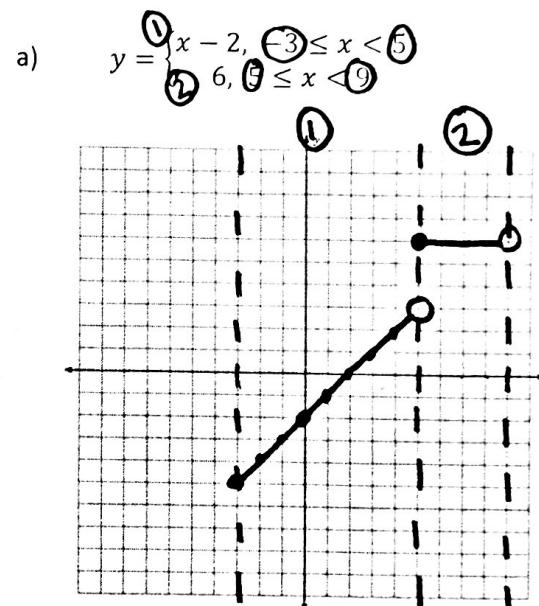
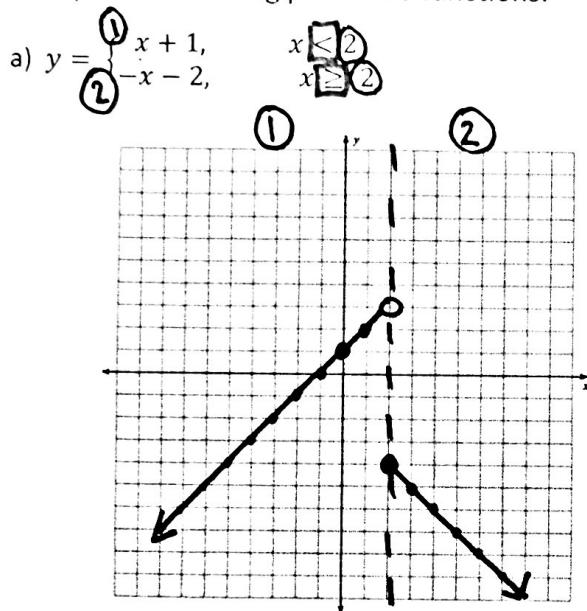


### Graphing Piecewise Functions

- 1) Put dotted lines on graph for each piece of domain
- 2) Label equations & graph sections
- 3) Graph each equation in its designated section
  - do not cross domain lines
  - use intervals for open holes & closed points

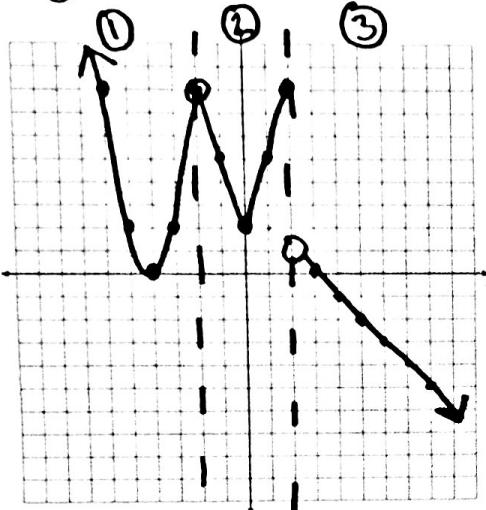
Open hole:  
 $\langle, \rangle$   
 Closed point:  
 $\leq, \geq$

3. Graph the following piecewise functions.



$$d) f(x) = \begin{cases} 2(x+4)^2, & x < -2 \\ 3|x| + 2, & -2 \leq x \leq 2 \\ -x + 4, & x > 2 \end{cases}$$

$\text{v: } (-4, 0)$   $a=2$



$$\begin{aligned} x_1 &= 2 \\ 3x &= 6 \\ x &= 2 \end{aligned}$$

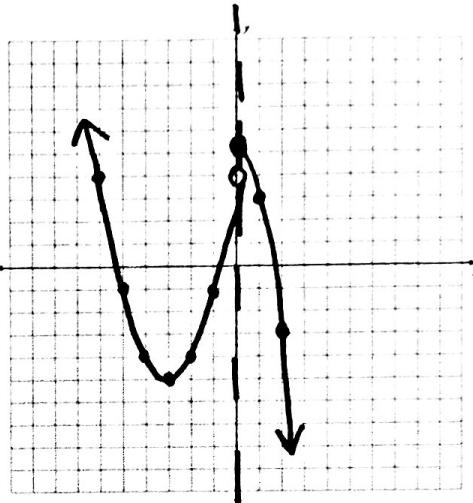
$$\text{v: } (0, 2) \quad a=3$$

$$e) f(x) = \begin{cases} (x+3)^2 - 5, & x < 0 \\ -2x^2 + 5, & x \geq 0 \end{cases}$$

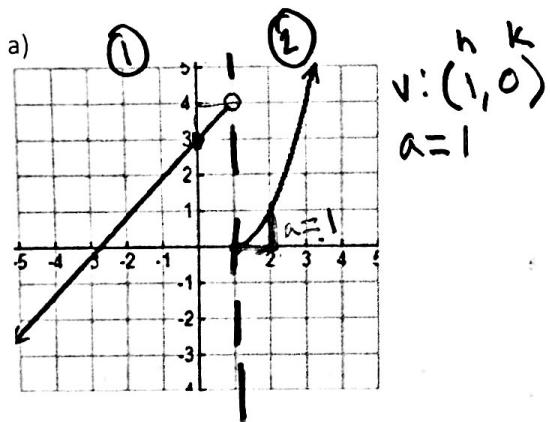
$\text{v: } (-3, -5)$   $a=1$

$$\begin{aligned} x_1 &= 1 \\ 3x &= 3 \\ x &= 1 \end{aligned}$$

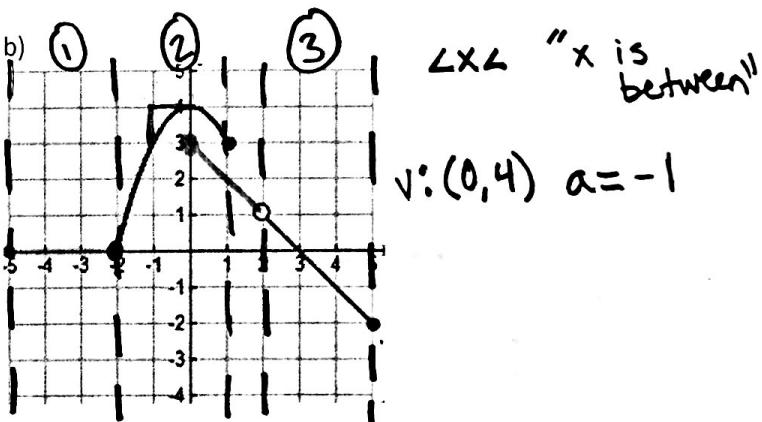
$$\begin{aligned} x_2 &= -2 \\ 3x &= -6 \\ x &= -2 \end{aligned}$$



2. Write an equation for the graphs below:



$$\begin{aligned} h &= 1 \\ k &= 0 \\ a &= 1 \end{aligned}$$



$\angle \angle \angle$  "x is between"

$$\text{v: } (0, 4) \quad a=-1$$

$$f(x) = \begin{cases} x + 3, & x < 1 \\ (x-1)^2, & x \geq 1 \end{cases}$$

$$f(x) = \begin{cases} 0, & -5 \leq x \leq -2 \\ -x^2 + 4, & -2 < x \leq 1 \\ -x + 3, & 2 < x \leq 5 \end{cases}$$

Horizontal line:  $y = c (\#)$

Line:  $y = mx + b$

$m = \text{slope}$   $b = y\text{-int}$

Parabola:  $y = a(x-h)^2 + k$   
 $(h, k) = \text{vertex}$   $a = \text{a-value}$

V-shape:  $y = a|x-h| + k$   
 $(h, k)$  is vertex  $a = \text{slope}$

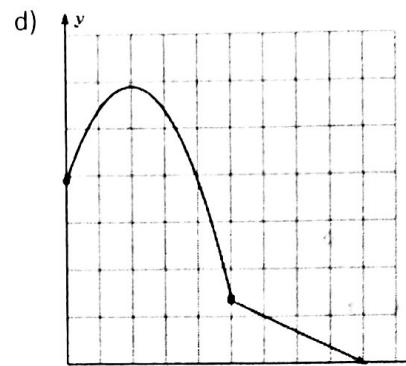
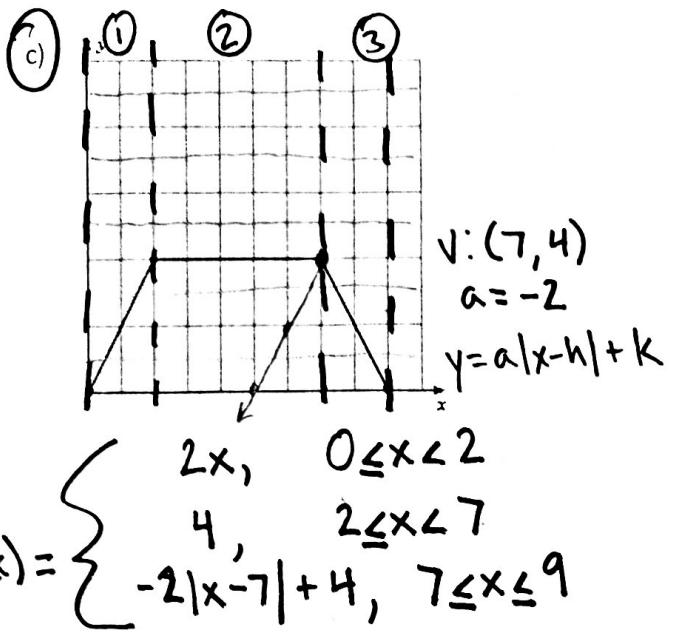
### Writing Piecewise Equations

1) Draw in dotted lines anywhere the graph changes shape

2) Write equation for each section of graph

3) Write intervals for each section  
• be careful of open holes & closed points

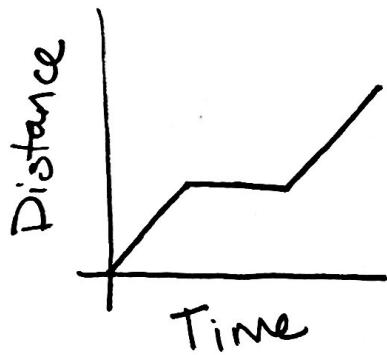
$<, >$        $\leq, \geq$



\* If two graph sections share a point, one of them must be open and one of them must be closed - you decide which

Example 4: Sketch a function that will model the following situations

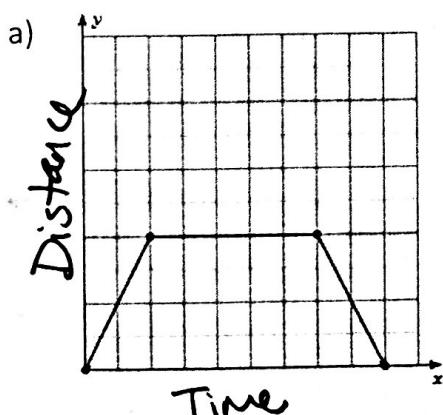
- a) John is taking his dog for a walk. He stops to tie his shoes, then continues walking.



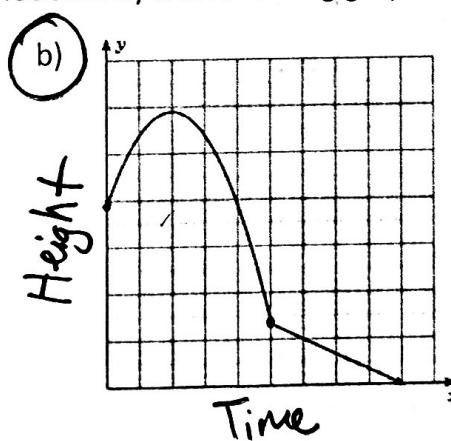
- b) A ball is thrown from standing height. It bounces 3 times (each bounce smaller than the one before) before rolling down the street.



Example 5: Describe the situation that could be modeled by the following graph:



Jane goes to the park, plays for a bit, then comes home.



A guy jumps from a cliff and hits the ground and rolls down the mountain.  
- Jackson Ward