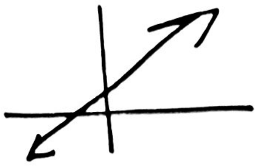
8.2 Exponential Functions and Average Rate of Change

1) Give an example of an equation that has a constant rate of change. Give your example as a graph and an equation.

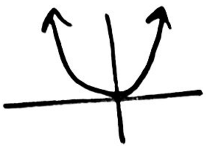
$y = 2x + 3$



Linear equations

2) Give an example of an equation that does not have a constant rate of change.

$y = x^2$



Non-linear equations

Because not all equations have a constant rate of change, we have to take a look at what we call the **average rate of change.** This looks at the rate of change of a function over a specified interval. These intervals refer to the x-coordinates.

Average Rate of Change Formula	Which also means...
For $[a, b]$ $\dfrac{f(b) - f(a)}{b - a}$	$\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\Delta y}{\Delta x} = \dfrac{\text{rise}}{\text{run}}$ Slope!

Steps for Average Rate of Change
1) Find y-coordinates of x's from interval 2) Label coordinats as (x_1, y_1) and (x_2, y_2) 3) Find slope between points

ex: Find the average rate of change for $y = x^2 - 2x + 1$ over the interval [-2, 3].

$x = -2$ $\quad y = (-2)^2 - 2(-2) + 1 = 9$ $\quad (-2, 9)$ $\quad x_1 \; y_1$

$x = 3$ $\quad y = (3)^2 - 2(3) + 1 = 4$ $\quad (3, 4)$ $\quad x_2 \; y_2$

$\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4 - 9}{3 - (-2)} = \dfrac{-5}{5} = \boxed{-1}$

3) Find the average rate of change for each function over the specified interval.

a. $y = 5x - 4$ over [-1, 6]

Linear: constant rate of change

$m = 5$ $\quad \boxed{5}$

b. $y = -x^2 + 5$ over [2, 5]

$x = 2$ $\quad y = -(2)^2 + 5 = 1$ $\quad (2, 1)$ $\quad x_1 \; y_1$

$x = 5$ $\quad y = -(5)^2 + 5 = -20$ $\quad (5, -20)$ $\quad x_2 \; y_2$

$\dfrac{-20 - 1}{5 - 2} = \dfrac{-21}{3} = \boxed{-7}$

c. $y = 2|x + 7| + 3$ over [-8, -4]

$x = -8$ $\quad y = 2|-8+7| + 3$
$= 2|-1| + 3 = 5$ $\quad (-8, 5)$ $\quad x_1 \; y_1$

$x = -4$ $\quad y = 2|-4+7| + 3$
$= 2|3| + 3 = 9$ $\quad (-4, 9)$ $\quad x_2 \; y_2$

$\dfrac{9 - 5}{-4 - (-8)} = \dfrac{4}{4} = \boxed{1}$

Exponential Functions

What is different about an exponential function? The variable is in the exponent.

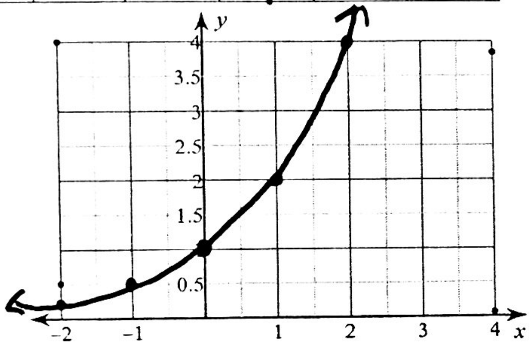
$y = a \cdot b^x$
a= initial value (y-int) x=0
b= growth/decay factor

We can have equations that either represent exponential growth or exponential decay. Let's see what happens when we graph:

To determine what makes our b value a growth factor or a decay factor, find each of the following

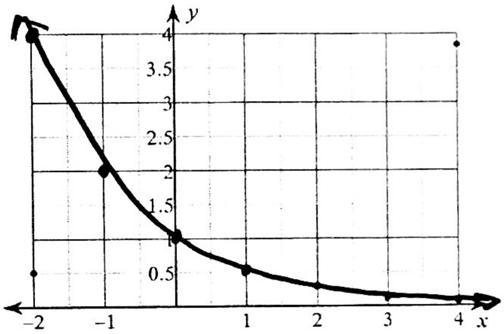
$f(x) = 2^x$ $= 1 \cdot 2^x$ $a = 1$ $b = 2$

x	-2	-1	0	1	2	3	4
y	0.25	0.5	1	2	4	8	16



$g(x) = (1/2)^x$ $= 1 \cdot (1/2)^x$ $a = 1$ $b = \frac{1}{2}$

x	-2	-1	0	1	2	3	4
y	4	2	1	.5	.25	0.125	.0625



Growth Factor	Decay factor
$b > 1$ ex: 2, 18.7, $\frac{4}{3}$	$b < 1$ ex: 0.37, $\frac{1}{3}$

4) Determine the initial value and the growth or decay factor for the equations below.

a. $y = 60 \cdot 1.41^x$
Initial: 60
Growth factor: 1.41

b. $y = 23 \cdot 0.67^x$
I: 23
DF: 0.67 (Decay factor)

c. $y = 525 \cdot 0.86^x$
I: 525
DF: 0.86

Another way to think about the growth/decay factor is to ask yourself what each value is being multiplied by each time.

5) Determine the growth/decay factor for each table below.

a.

x	-2	-1	0	1	2	3
y	0.11	0.33	1	3	9	27

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x=0
I: 1
GF: 3

$y = 1 \cdot 3^x$ or $\boxed{y = 3^x}$

b.

x	-2	-1	0	1	2
y	32	8	2	0.5	0.13

÷4 ÷4 ÷4 ÷4

Dividing by 4 is the same as multiplying by $\frac{1}{4}$

I: 2
DF: $\frac{1}{4}$

$y = 2\left(\frac{1}{4}\right)^x$

The growth/decay **RATE** is the percent change between each output value of the function.

Let's see if you can figure out how to find the growth/decay rate by looking at the following examples:

$f(x) = 3 \cdot 1.23^x$ Growth rate: 23%

$g(x) = 0.5 \cdot 1.64^x$ Growth rate: 64%

$h(x) = 2\left(\frac{3}{4}\right)^x$ Decay rate: 25%

$k(x) = 0.4 \cdot 0.39^x$ Decay rate: 61%

Growth Rate	Decay rate
Growth factor → b − 1	1 − b ← Decay factor

6) Identify the initial value, the growth or decay factor, and the growth or decay rate for each of the functions below.

a. $f(x) = 4 \cdot 0.78^x$

b. $y = 5 \cdot 1.47^x$

c. $g(t) = 0.6 \cdot 1.19^t$
I: 0.6
GF: 1.19
GR: 19%

d. $y = 1.5 \cdot 0.36^x$
I: 1.5
DF: 0.36
DR: $1 - 0.36 = 0.64$
$\boxed{64\%}$

e. $h(x) = 3\left(\frac{2}{5}\right)^x$

f. $k(x) = 2 \cdot 2^x$

7) Find the growth or decay rate factor for the functions below and state the growth or decay rate.

a. $f(x) = 1.05^{4x} = (1.05^4)^x = 1.22^x = 1 \cdot 1.22^x$
I: 1
GF: 1.22 GR: 22%

b. $h(t) = (0.68^3)^t = 0.31^t = 1 \cdot 0.31^t$
I: 1 DF: 0.31
DR: 0.69 $\boxed{69\%}$

c. $y = 1.46^{3x}$

8) Find a bank account balance if the account starts with $100, has an annual growth rate of 4%, and the money left in the account for 12 years.

GR = .04
GF = 1.04

$y = a \cdot b^x$ ← time
Initial → a; Growth/decay factor → b

a = 100
b = 1.04

$y = 100 \cdot 1.04^{12} \approx \boxed{\$160.10}$

9) In 1985, there were 285 cell phone subscribers in the small town of Centerville. The number of subscribers increased by 75% per year after 1985. How many cell phone subscribers were in Centerville in 1994?

I: 285 (a) 1994 − 1985 = 9 (x)
GR: 0.75
GF: 1.75 (b)

$y = 285 \cdot 1.75^9 \approx 43871.99 \approx$ $\boxed{43{,}872 \text{ subscribers}}$

10) Each year the local country club sponsors a tennis tournament. Play starts with 128 participants. During each round, half of the players are eliminated. How many players remain after 5 rounds?

a = 128 x = 5
DR = 50% = .5
DF = 1 − .5 = .5

$y = 128 \cdot 0.5^5 = \boxed{4 \text{ players}}$