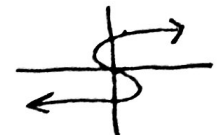# 8.2 Functions and Average Rate of Change

You have been hearing about functions for a good chunk of your math career, but do you actually know what a function is?

Mathematical Definition	Translation	What it looks like
A function is a relationship where each input has exactly one output.	When you plug something into an equation, you only get one answer (vending machine)	

Function ｜ Not a function
* Vertical line test

Here is what this looks like in terms of equations:

Equation	Function
$y = 2x - 5$	$f(x) = 2x - 5$
$y = x^2 - 3x + 10$	$g(x) = x^2 - 3x + 10$
For $y = 2x - 5$, <u>find y when x=3</u> plug in 3	For $f(x) = 2x - 5$, <u>find $f(3)$</u> plug in 3
To find the vertex from standard form: $\left(\frac{-b}{2a}, \text{plug it in}\right)$	$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

(x, y)
input ↗ ↖ output
$(x, f(x))$

How do $f(x)$ and y relate to each other?

They are the same thing!

1) For $f(x) = 3x - 7$, evaluate the function for the following:

a. $f(-5) = 3(-5) - 7$
$(-5, -22)$ $= -15 - 7 = \boxed{-22}$

b. $f(7) = 3(7) - 7$
$= 21 - 7 = \boxed{14}$

c. $f(0) = 3(0) - 7$
$= 0 - 7 = \boxed{-7}$

2) For $f(x) = x^2 - 2x + 5$, evaluate the function for the following:
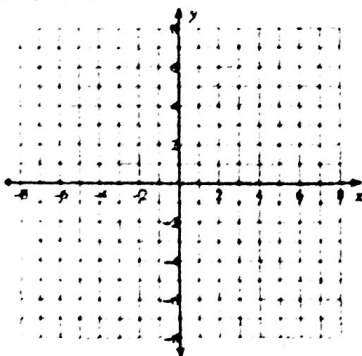
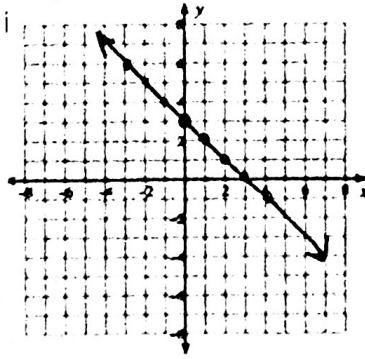d. $f(-4) = (-4)^2 - 2(-4) + 5$
$= 29$

e. $f(-1)$

f. $f(6)$
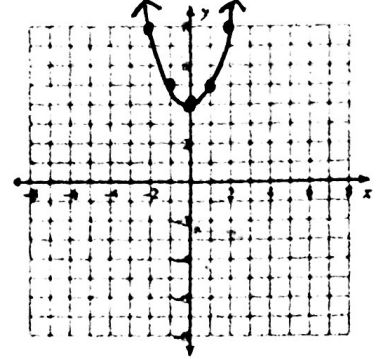
3) Graph the following functions.

g. $f(x) = 2x - 1$

h. $f(x) = -x + 3$

j. $f(x) = x^2 + 4$

You are already familiar with the concept of "average rate of change". When working with lines, the average rate of change was the slope. Let's review how to do this:

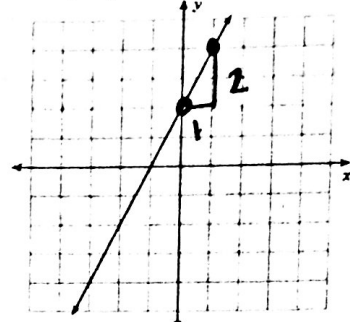| SLOPE FORMULA: |
|---|

$$\frac{rise}{run}$$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

a) Find the slope between the points: $(1, 2)$ and $(-3, 4)$

$$-4 \left\langle \begin{array}{c|c} x & y \\ 1 & 2 \\ -3 & 4 \end{array} \right\rangle 2$$

$$\frac{2}{-4} = \boxed{-\frac{1}{2}}$$

b) Find the slope of the line on the graph below:
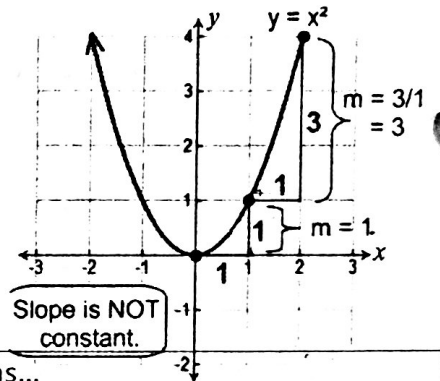

$$\frac{2}{1} = \boxed{2}$$

A linear function has a __constant__ rate of change. What does that mean?

It is always the same.

NON-LINEAR FUNCTIONS:

When working with non-linear functions, the "average rate of change" is not constant.

The process of computing the "average rate of change", however, remains the same as was used with straight lines:
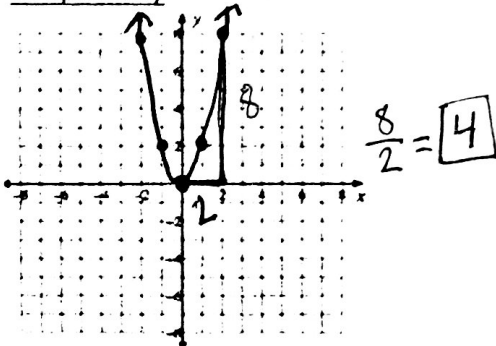

Slope is NOT constant.

Average Rate of Change Formula	Which also means...
$$\frac{f(b) - f(a)}{b - a}$$	$\frac{y_2 - y_1}{x_2 - x_1}, \quad \frac{rise}{run},$ Find the slope

Example 1: For the function $f(x) = 2x^2$, find the average rate of change on the interval $[0, 2]$.

(x-values)

Graphically

$$\frac{8}{2} = \boxed{4}$$

Algebraically

$$+2 \left\langle \begin{array}{c|c} x & f(x) \\ 0 & 0 \\ 2 & 8 \end{array} \right\rangle +8$$

$$\frac{8}{2} = \boxed{4}$$

$$f(0) = 2(0)^2 = 2(0) = 0$$
$$f(2) = 2(2)^2 = 2(4) = 8$$

PEMDAS

As you choose different intervals, the average rate of change will differ (Unless the function is a line).

Example 2: Using $f(x) = 2x^2$, find the average rate of change for the following intervals:

a) $[-1, 2]$

$f(-1) = 2(-1)^2$
$\quad = 2(1) = 2$

$f(2) = 2(2)^2 = 2(4) = 8$

$3 < \begin{array}{c|c} x & y \\ \hline -1 & 2 \\ 2 & 8 \end{array} > 6$

$\frac{6}{3} = \boxed{2}$

b) $[-3, 3]$

$f(-3) = 2(-3)^2 = 2(9) = 18$
$f(3) = 2(3)^2 = 2(9) = 18$

$\frac{0}{6} = \boxed{0}$

$6 < \begin{array}{c|c} x & y \\ \hline -3 & 18 \\ 3 & 18 \end{array} > 0$

Example 3: Using the function: $g(x) = -2x^2 + 2x$, find the average rate of change over the following intervals:

a) $[4, 8]$

$g(4) = -2(4)^2 + 2(4) = -24$
$g(8) = -2(8)^2 + 2(8) = -112$

$4 < \begin{array}{c|c} x & y \\ \hline 4 & -24 \\ 8 & -112 \end{array} > -88$

$\Big| \frac{-88}{4} = \boxed{-22}$

b) $[-6, 0]$

c) $[-4, 8]$

Example 4:

a) Suppose your family decides to drive down to St. George. It is about 275 miles from Highland to St. George and according to Google Maps it will take approximately 3 hours and 40 minutes to get there. At what constant speed would you have to drive in order to arrive in St. George in exactly 3 hours and 40 minutes?

b) Your family decides to leave at 8AM on Saturday morning and your dad drives. Because it's a long drive and you get bored, for the first couple of hours you decide to note the time that has passed and the distance you have travelled. The times are depicted in the table below:

Time	Distance
8:15	5
8:20	10
8:30	20
8:50	45
9:05	64
9:35	94
9:45	108
9:50	114
10:00	126

What was your average rate of change from:

a) 8:00-8:15?

b) 8:50 to 9:45

c) 8:00 and 10:00?