Unit 6.4: Binomial Radical Expressions

Like Radicals are radical expressions that have the same index and radicand. This basically means that we [illegible] going to combine like radicals just like we combine like terms in a regular expression.

Like Radicals with Numbers

$\sqrt{2} + 3\sqrt{2} = 4\sqrt{2}$

$\sqrt[3]{7} - 5\sqrt[3]{7} = -4\sqrt[3]{7}$

Like Radicals with Variables

$\sqrt{5xy} + 8\sqrt{5xy} = 9\sqrt{5xy}$

$\sqrt[3]{9x^2y} - 8\sqrt[3]{9x^2y} = -7\sqrt[3]{9x^2y}$

We treat like radicals (same index & inside) like we treat like terms.

Adding and Subtracting Radical Expressions:

PRACTICE:

1. $7\sqrt[3]{5} - 4\sqrt[3]{5}$

$\boxed{3\sqrt[3]{5}}$

2. $7\sqrt[3]{x^2} - 2\sqrt[3]{x^2}$

$\boxed{5\sqrt[3]{x^2}}$

3. $17\sqrt[5]{3x} + 15\sqrt[5]{3x}$

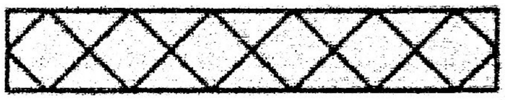
$\boxed{32\sqrt[5]{3x}}$

4. $3\sqrt{5x^2} + 2\sqrt{5x^2}$

$5\sqrt{5x^2} = \boxed{5|x|\sqrt{5}}$

$x \; x$

5. The design of a garden path uses stone pieces shaped as squares with a side length of 15 in. Find the length of the path.



6. $\sqrt{12} + \sqrt{75} - \sqrt{3}$

Simplify first!

$4 \; 3 \quad 3 \; 25$

$2 \; 2 \quad 5 \; 5$

$2\sqrt{3} + 5\sqrt{3} - \sqrt{3}$

$= \boxed{6\sqrt{3}}$

Multiplying Binomial Radical Expressions

- You can distribute just like you do when you multiply any binomials

What is the product of each radical expression?

7. $(3 - \sqrt{7})(5 + \sqrt{7})$

$\sqrt{7} \cdot \sqrt{7} = \sqrt{49} = 7$

$15 + 3\sqrt{7} - 5\sqrt{7} - 7$

$\boxed{8 - 2\sqrt{7}}$

8. $(4 + 2\sqrt{2})(5 + 4\sqrt{2})$

$2\sqrt{2} \cdot 4\sqrt{2} = 8 \cdot 2 = 16$

$20 + 16\sqrt{2} + 10\sqrt{2} + 16$

$\boxed{36 + 26\sqrt{2}}$

It's a lot less work to simplify first!

9. $(\sqrt{3}+\sqrt{5})^2$

$(\sqrt{3}+\sqrt{5})(\sqrt{3}+\sqrt{5})$

$3+\sqrt{15}+\sqrt{15}+5 = \boxed{8+2\sqrt{15}}$

10. $(5-\sqrt{9})(5+\sqrt{7})$

$25+5\sqrt{7}-5\sqrt{9}-\sqrt{63}$

or...

$\sqrt{9}=3 \quad (5-3)(5+\sqrt{7}) = 2(5+\sqrt{7}) = \boxed{10+2\sqrt{7}}$

11. $(3-\sqrt{8})(3+\sqrt{8})$ $\quad \sqrt{8}\cdot\sqrt{8}=8$

$9+3\sqrt{8}-3\sqrt{8}-8$

$=\boxed{1}$ $\quad 3\sqrt{8}-3\sqrt{8}=0$

12. $(4+2\sqrt{3})(4-2\sqrt{3})$ $\quad 2\sqrt{3}\cdot 2\sqrt{3} = 4\cdot 3 = 12$

$16-\cancel{8\sqrt{3}}+\cancel{8\sqrt{3}}-12$

$=\boxed{4}$

Which questions above had rational solutions? What is going on in the binomials that had a rational product?

Same numbers, opposite signs between them

Conjugate Pairs: $\pm$ pair from a $\sqrt{\ }$

Multiplying by the conjugate gets rid of the root, giving us a rational answer

We dealt a little bit with conjugate pairs in unit 3. We multiplied by a conjugate to get rid of an i. Turns out we can multiply by a conjugate to get rid of a square root as well. **We only need to do this if there is a radical binomial in the denominator.**

$\frac{3\sqrt{2}}{\sqrt{5}-\sqrt{2}} \cdot \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}} = \frac{3\sqrt{10}+6}{3}$ *

$=\boxed{\sqrt{10}+2}$

$3\sqrt{2}(\sqrt{5}+\sqrt{2}) = 3\sqrt{10}+6$ $\quad 3\sqrt{2}\cdot\sqrt{2} = 3\cdot 2 = 6$

$(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2}) = 5+\cancel{\sqrt{10}}-\cancel{\sqrt{10}}-2 = 3$

Rationalize the denominator of each expression.

13. $\frac{4}{1+\sqrt{3}} \cdot \frac{1-\sqrt{3}}{1-\sqrt{3}} = \frac{4-4\sqrt{3}}{-2}$ *

$=\boxed{-2+2\sqrt{3}}$

$(1+\sqrt{3})(1-\sqrt{3})$

$1-\cancel{\sqrt{3}}+\cancel{\sqrt{3}}-3$

-2

*The three numbers outside the root reduce all together or not at all

14. $\frac{3+\sqrt{8}}{2-2\sqrt{8}} \cdot \frac{2+2\sqrt{8}}{2+2\sqrt{8}} = \frac{22+16\sqrt{2}}{-28}$ * $= \boxed{\frac{-11-8\sqrt{2}}{14}}$

$(3+\sqrt{8})(2+2\sqrt{8})$ $\quad \sqrt{8}\cdot 2\sqrt{8} = 2\cdot 8 = 16$

$6+6\sqrt{8}+2\sqrt{8}+16 = 22+8\sqrt{8}$

$= 22+16\sqrt{2}$

$\sqrt{8}$: $2 \wedge 4$, 2

$(2-2\sqrt{8})(2+2\sqrt{8})$ $\quad 2\sqrt{8}\cdot 2\sqrt{8} = 4\cdot 8 = 32$

$4+\cancel{4\sqrt{8}}-\cancel{4\sqrt{8}}-32 = -28$