6.3 Multiplying and Dividing Radicals

For each of the exponent properties below, demonstrate the reasoning as to why each property works.

Product of Powers
Simplify the following exponents:

$6^3 \cdot 6^5 = 6^8$
$(6\cdot6\cdot6)\cdot(6\cdot6\cdot6\cdot6\cdot6)$

$4^7 \cdot 4^2 = 4^9$
$(4\cdot4\cdot4\cdot4\cdot4\cdot4\cdot4)\cdot(4\cdot4)$

Quotient of Powers
Simplify the following exponents:

$\frac{2^7}{2^4} = \frac{2\cdot\cancel{2}\cdot\cancel{2}\cdot\cancel{2}\cdot\cancel{2}\cdot2\cdot2}{\cancel{2}\cdot\cancel{2}\cdot\cancel{2}\cdot\cancel{2}} = \boxed{2^3}$

$\frac{8^5}{8^2} = \frac{\cancel{8}\cdot\cancel{8}\cdot8\cdot8\cdot8}{\cancel{8}\cdot\cancel{8}} = \boxed{8^3}$

Power of Powers
Simplify the following exponents:

$(2^3)^3 = 2^9$
$(2^3)\cdot(2^3)\cdot(2^3)$

$(5^2)^4 = 5^8$
$(5^2)\cdot(5^2)\cdot(5^2)\cdot(5^2)$

For the examples below, see if you can figure out the rule on how to simplify the expression. Write your thinking in the space that follows the examples.

$2^2 \cdot 3^2 = 6^2$
$2\cdot2\cdot3\cdot3 = (2\cdot3)\cdot(2\cdot3) = 6\cdot6$

$4^3 \cdot 5^3 = 20^3$
$4\cdot4\cdot4\cdot5\cdot5\cdot5 = (4\cdot5)\cdot(4\cdot5)\cdot(4\cdot5)$
$= 20\cdot20\cdot20$

Ideas:

We can combine exponents when they have the same base, but turns out we can also combine bases when they have the same exponent.

1. Simplify the following expressions.
   a. $5^7 \cdot 8^7 = \boxed{40^7}$
   same exponent

   b. $4^7 \cdot 4^3 = \boxed{4^{10}}$
   same base

   c. $2^{-3} \cdot 2^9$
   same base

   d. $3^{\frac{1}{2}} \cdot 7^{\frac{1}{2}} = 21^{\frac{1}{2}}$
   $\sqrt{3}\cdot\sqrt{7} = \sqrt{21}$
   ↑
   Since rational exponents represent a root, we can apply this same rule to roots.

   e. $6^{\frac{4}{5}} \cdot 5^{\frac{4}{5}} = \boxed{30^{\frac{4}{5}}}$
   same exponent

Recall that rational exponents mean that you are taking a root of some kind. This means that these exponent properties apply to radicals.

Property	Definition	Example
Product Property of Radicals	When roots are the same index, you can multiply the insides & simplify together	$\sqrt[3]{4x^2}\cdot\sqrt[3]{6x} = \sqrt[3]{24x^3}$ 8 3 x x x 2 2 2 $\boxed{2x\sqrt[3]{3}}$
Quotient Property of Radicals	Take the root of a fraction by taking the root of top & bottom separately	$\sqrt{\frac{25x^2}{9}} = \frac{\sqrt{25x^2}}{\sqrt{9}} = \boxed{\frac{5\lvert x\rvert}{3}}$

2. Simplify each product or quotient. Be sure to rationalize all denominators.

a) $\sqrt[3]{-12}\cdot\sqrt[3]{-18}$

$= \sqrt[3]{216} = \boxed{6}$

b) $\sqrt{81x^3y^2}\cdot\sqrt{27y^4}$

9 9 x x x y y 9 3 3 3 y y y y

$\boxed{27\lvert xy^3\rvert\sqrt{3x}}$

c) $\sqrt[4]{20x^2}\cdot\sqrt[4]{12x^5}$ ←

Because we would break it down after multiplying anyway we can break them down separately and treat it as one factor tree

d) $3\sqrt[4]{18a^9}\cdot\sqrt[4]{6ab^2}$ – bb

9 2 3 2 aaaa aaaa aa

3 3

$3\cdot3a^2\sqrt[4]{4a^2b^2}$

$\boxed{9a^2\sqrt[4]{4a^2b^2}}$

e) $\sqrt{\frac{15}{16}} = \frac{\sqrt{15}}{\sqrt{16}} = \boxed{\frac{\sqrt{15}}{4}}$

$\sqrt{15}$

3 5

f) $\sqrt{\frac{36y^4}{3}} = \sqrt{12y^4}$

4 3 y y y y

2 2

$\boxed{2y^2\sqrt{3}}$

*Simplify fraction first

g) $\frac{\sqrt{\cancel{18}^{9}\cancel{x^5}^{x^2}}}{\sqrt{\cancel{2x^3}}} = \sqrt{9x^2} = \boxed{3\lvert x\rvert}$

h) $\sqrt[3]{\frac{50xy^3}{.5}}$

i) $\sqrt{\frac{\cancel{22}^{11}\cancel{x^3}\cancel{y^3}}{\cancel{10}_{5}\cancel{x^4}\cancel{y}}} = \sqrt{\frac{11y}{5x}}$

$= \frac{\sqrt{11y}}{\sqrt{5x}}\cdot\frac{\sqrt{5x}}{\sqrt{5x}} = \boxed{\frac{\sqrt{55xy}}{5x}}$

Rationalizing Denominators

Multiply so you have as many as the index of the denominator, then the root will cancel.

ex: $\frac{2}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \boxed{\frac{2\sqrt{5}}{5}}$

$(\sqrt{5})^2 = 5$

ex: $\frac{3}{\sqrt[3]{4}}\cdot\frac{\sqrt[3]{4}}{\sqrt[3]{4}}\cdot\frac{\sqrt[3]{4}}{\sqrt[3]{4}} = \frac{3\sqrt[3]{16}}{4} = \frac{6\sqrt[3]{2}}{4}$

$= \boxed{\frac{3\sqrt[3]{2}}{2}}$

$(\sqrt[3]{4})^3 = 4$