6.2 Properties of Radicals

The first thing we are going to talk about is called the **principal square root.** Basically when you are asked for a square root, it is assumed that the person doing the asking wants the positive square root, which is what we call the principal square root.

$\sqrt{25}$ vs. $x^2 = 25$

5 $x = \pm 5$

Why do you need $\pm$ when taking a square root to solve an equation?

Because you can plug in the positive or negative & have the sides equal each other

Cube Roots

A negative inside of a square root leads to an imaginary answer. That is not the case with cube roots. If you cube a negative number, the result is negative. This means that we can have negative cube roots that are still real numbers.

a. $\sqrt[3]{-27} = -3$ b. $\sqrt[3]{64} = 4$ c. $\sqrt[3]{-343} = -7$

d. $\sqrt[3]{-1000} = -10$ e. $\sqrt[3]{125} = 5$

Math → 4: $\sqrt[3]{}$

Imaginary answers come from a negative under a root with an even index.

Simplifying Radicals

Tips to simplify radicals

- the index tells you how big the groups need to be to come out of the radical
  ex: $\sqrt[4]{}$ → groups of 4

a) $\sqrt[3]{132} = \sqrt[3]{132}$

2 | 66
2 | 33
3 | 11

No groups of 3, so it doesn't simplify

b) $\sqrt{392}$

2 | 196
2 | 98
2 | 49
7 | 7

$14\sqrt{2}$

c) $3\sqrt[4]{112}$

2 | 56
2 | 28
2 | 14
2 | 7

$3 \cdot 2\sqrt[4]{7}$

$6\sqrt[4]{7}$

d) $6\sqrt[3]{225}$

e) $3\sqrt[4]{1215}$

f) $2\sqrt[4]{400}$

4 | 100
2 | 50
2 | 25
5 | 5

$2 \cdot 2\sqrt[4]{25}$

$4\sqrt[4]{25}$

Variables change things a bit because a variable can be positive or negative. Remember: when you are asked to find a square root, you are being asked to find the principal square root, so it is possible that you will need to restrict your answer to only positive numbers. To do this, you use absolute values in your solution.

When to use absolute value signs *All conditions must be met in order to need absolute value signs

1) Real number (no imaginary answers)
2) Even index
3) Odd exponent after coming out of root

* Absolute value signs are only needed on variables

a) $\sqrt{128x^4}$

$2 \cdot 64$, $64 = 8 \cdot 8$, $x^4 = x \cdot x \cdot x \cdot x$

$8x^2\sqrt{2}$

Real ✓
Even index ✓
Odd exponent X
No absolute value needed

b) $\sqrt[3]{72x^4}$

c) $2\sqrt{12x^3}$

$12 = 4 \cdot 3$, $4 = 2 \cdot 2$, $x^3 = x \cdot x \cdot x$

$2 \cdot 2x\sqrt{3x} = 4|x|\sqrt{3x}$

Real ✓
Even index ✓
Odd exponent ✓
Need absolute value.

d) $3\sqrt{175x^3y^2}$

e) $6\sqrt{24x^4y^2}$

$24 = 8 \cdot 3$, $8 = 4 \cdot 2$, $4 = 2 \cdot 2$, $x^4 = x \cdot x \cdot x \cdot x$, $y^2 = y \cdot y$

$6 \cdot 2x^2y\sqrt{6}$

$12x^2|y|\sqrt{6}$

f) $\sqrt[3]{64x^6}$

$64 = 16 \cdot 4$, $16 = 4 \cdot 4$, $x^6 = x \cdot x \cdot x \cdot x \cdot x \cdot x$

$4x^2$

Real ✓
Even index X
No absolute value needed