6.1: Properties of Exponents

Rational exponents are a way of representing radical expressions. The denominator of the fraction signifies what kind of root you are taking (index). The numerator signifies the exponent of the base inside the root.

Ex: $3^{\frac{1}{2}} = \sqrt{3}$ $\quad m^{\frac{1}{5}} = \sqrt[5]{m}$ $\quad y^{\frac{2}{3}} = \sqrt[3]{y^2}$ $\quad 2x^{\frac{2}{5}} = 2\sqrt[5]{x^2}$

1) Convert each fractional exponent to a radical expression or vice versa.

a. $x^{\frac{4}{5}}$ $\sqrt[5]{x^4}$ or $(\sqrt[5]{x})^4$

b. $3a^{\frac{1}{3}}$ $3\sqrt[3]{a}$

c. $(3a)^{\frac{2}{3}}$ $\sqrt[3]{(3a)^2}$ or $(\sqrt[3]{3a})^2$

d. $\sqrt{5}$ $5^{\frac{1}{2}}$

e. $\cancel{\sqrt[4]{2x^5}}$

f. $\sqrt[3]{(6x)^4}$ $(6x)^{\frac{4}{3}}$

Review of Exponent Properties

Property	Rule	Example
Zero Property	Anything to the 0 power is 1	• $a^0 = 1$ • $12^0 = 1$
Negative Exponent Property	Gives the reciprocal (flips) fraction, exponent becomes positive once flipped. Term in numerator moves to denominator or vice versa.	• $\left(\frac{1}{a}\right)^{-1} = \left(\frac{a}{1}\right)^1 = a$ • $(2a)^{-2} = \left(\frac{1}{2a}\right)^2 = \boxed{\frac{1}{4a^2}}$ • $2a^{-3} = 2\left(\frac{1}{a}\right)^3 = \boxed{\frac{2}{a^3}}$
Product of Powers Property	Add exponents, multiply coefficients normally	• $a^4 \cdot a^3 = a^7$ • $5a^2 \cdot 2a^9 = 10a^{11}$
Quotient of Powers Property	Subtract exponents, divide coefficients normally	• $\frac{a^7}{a^2} = a^4$ • $\frac{6a^{10}}{2a^{-1}} = 3a^{11}$
Power of a Power Property	Multiply exponents • exponent applies to everything	• $(a^3)^2 = a^6$ • $(2x^2)^5 = 2^5x^{10} = \boxed{32x^{10}}$

REMEMBER: We can only combine exponents when the bases are the same.

*Feel free to use calculator to add/subtract fractions

2) Simplify. Your answer should contain only positive exponents.

a. $(m^{\frac{1}{2}})^{\frac{3}{1}}$

$M^{\frac{3}{2}}$

b. $(m^{\frac{1}{2}} \cdot m^{\frac{2}{5}})^4$ $\quad \frac{1}{2}+\frac{2}{5}=\frac{9}{10}$

$(M^{\frac{9}{10}})^{\frac{4}{1}}$

$M^{\frac{36}{10}} = \boxed{M^{\frac{18}{5}}}$

c. $\dfrac{x^{\frac{3}{5}}}{x^{\frac{1}{2}}}$ $\quad \frac{3}{5}-\frac{1}{2}=\frac{1}{10}$

$\boxed{X^{\frac{1}{10}}}$

d. $\dfrac{3x^{\frac{1}{4}}}{x^{\frac{1}{2}}}$ $\quad \frac{1}{4}-\frac{1}{2}=-\frac{1}{2}$

$3X^{-\frac{1}{2}} = \boxed{\dfrac{3}{X^{\frac{1}{2}}}}$

e. $(2y^{\frac{1}{2}} \cdot 6y^{\frac{2}{5}})^4$

$(12y^{\frac{9}{10}})^4$

$\boxed{12^4 y^{\frac{18}{5}}}$

f. $\dfrac{(4x^{\frac{1}{4}})^2}{x^{\frac{1}{2}}} = \dfrac{4^2 X^{\frac{2}{4}}}{X^{\frac{1}{2}}}$

$\dfrac{16\cancel{x^{\frac{1}{2}}}}{\cancel{x^{\frac{1}{2}}}} = \boxed{16}$

You will be asked to simplify expressions that have different bases. When this happens, you must change the bases to be the same before you can start combining their exponents.

3) Simplify each of the following expressions.

a. $5^4 \cdot 25^2$

$5^4 \cdot (5^2)^2$

$5^4 \cdot 5^4 = \boxed{5^8}$

b. $8^{\frac{3}{5}} \cdot 2^{\frac{1}{5}}$

$(2^3)^{\frac{3}{5}} \cdot 2^{\frac{1}{5}}$

$2^{\frac{9}{10}} \cdot 2^{\frac{1}{5}} = 2^{\frac{10}{5}} = 2^2 = \boxed{4}$

c. $4^{\frac{1}{4}} \cdot 16^{\frac{2}{3}}$

d. $8^{\frac{5}{6}} \cdot 16^{\frac{1}{3}}$

e. $9^{\frac{3}{2}} \cdot 27^{\frac{6}{5}}$

$(3^2)^{\frac{3}{2}} \cdot (3^3)^{\frac{6}{5}}$

$3^3 \cdot 3^{\frac{18}{5}} = \boxed{3^{\frac{33}{5}}}$

f. $25^{\frac{5}{2}} \cdot 125^{\frac{1}{4}}$

Bases have to be the same before combining exponents.