4.3: Solving by Completing the Square

...st time we looked at a perfect square quadratic. They look like:

$$f(x) = (x+3)(x+3) \textbf{ OR } f(x) = (x+3)^2$$

Example 1: Distribute the following expressions. *Remember that something squared means it is multiplied by itself

a) $(x+5)^2$

$(x+5)(x+5)$

$x^2+5x+5x+25$

$\boxed{x^2+10x+25}$

b) $(x+3)^2$

$(x+3)(x+3)$

$x^2+3x+3x+9$

$\boxed{x^2+6x+9}$

c) $(x-7)^2$

$(x-7)(x-7)$

$x^2-7x-7x-49$

$\boxed{x^2-14x-49}$

Notice that by distributing you have put each expression in standard form, ax^2+bx+c

How does the number in parentheses relate to the *b* coefficient?

The number in parentheses is half of the middle coefficient (b)

How does the number in parentheses relate to the constant, *c*?

If you square the number in parentheses, you get the constant (c)

...s kind of relationship always exists with a perfect square quadratic. We will use this to do a process called completing the square, which makes any expression into a perfect square quadratic.

Example 2: Find the value of c that completes the square.

a) $x^2 - 6x + c$ $\boxed{c=9}$

$-6 \div 2 = -3$

$(-3)^2 = 9$ $\to (x-3)^2$

b) $x^2 + 36x + c$ $\boxed{c=324}$

$36 \div 2 = 18 \to (x+18)^2$

$(18)^2 = 324$

c) $x^2 - 20x + c$ $\boxed{c=100}$

$-20 \div 2 = -10 \to (x-10)^2$

$(-10)^2 = 100$

Example 2: Rewrite each equation by completing the square

a) $x^2 + 10x - 11 = 0$

Notice this is not a perfect square since the two properties we know are not met.

b) $x^2 - 8x - 9 = 0$

$-8 \div 2 = -4$

$(-4)^2 = 16$

c) $x^2 + 20x + 36 = 0$

d) $x^2 - 12x - 85 = 0$

The process of completing the square makes any quadratic into a perfect square. We do this process so we can solve by taking a square root.

Steps to Complete the Square:
1) Move constant to other side of equation 2) Find the value that completes the square & add to both sides • Divide middle coefficient by 2 & square it 3) Rewrite as $(\quad)^2$

Example 4: Solve each equation by completing the square.

a) $x^2 + 8x + 15 = 0$

$\quad\quad\quad\quad\quad -15 \quad -15$

1. $x^2 + 8x = -15$

2. $x^2 + 8x + 16 = -15 + 16$

$8 \div 2 = 4$

$(4)^2 = 16$

3. $(x+4)^2 = 1$

$\sqrt{(x+4)^2} = \sqrt{1}$

$x + 4 = \pm 1$

$-4 \quad -4$

$x = -4 \pm 1$

$-4 + 1 = \boxed{-3}$

$-4 - 1 = \boxed{-5}$

b) $x^2 + 18x + 56 = 0$

When we find the value that completes the square, we need to add it to both sides.

After successfully completing the square, solve by taking a square root.

c) $x^2 - 12x - 85 = 0$

$\quad\quad\quad\quad\quad +85 \quad +85$

1. $x^2 - 12x = 85$

2. $x^2 - 12x + 36 = 85 + 36$

$-12 \div 2 = -6$

$(-6)^2 = 36$

3. $(x-6)^2 = 121$

$\sqrt{(x-6)^2} = \sqrt{121}$

$x - 6 = \pm 11$

$x = 6 \pm 11$

$6 + 11 = \boxed{17}$

$6 - 11 = \boxed{-5}$

d) $x^2 + 6x + 58 = 0$

$\quad\quad\quad\quad\quad -58 \quad -58$

$x^2 + 6x + 9 = -58 + 9$

$6 \div 2 = 3$

$3^2 = 9$

$(x+3)^2 = -49$

$\sqrt{(x+3)^2} = \sqrt{-49}$

$x + 3 = \pm 7i$

$x = -3 \pm 7i$

$\boxed{-3+7i}$

$\boxed{-3-7i}$