

4.3: Solving by Completing the Square

At time we looked at a perfect square quadratic. They look like:

$$f(x) = (x + 3)(x + 3) \text{ OR } f(x) = (x + 3)^2$$

Example 1: Distribute the following expressions.

*Remember that something squared means it is multiplied by itself

a) $(x + 5)^2$

$$(x+5)(x+5)$$

$$x^2 + 5x + 5x + 25$$

$$\boxed{x^2 + 10x + 25}$$

b) $(x + 3)^2$

$$(x+3)(x+3)$$

$$x^2 + 3x + 3x + 9$$

$$\boxed{x^2 + 6x + 9}$$

c) $(x - 7)^2$

$$(x-7)(x-7)$$

$$x^2 - 7x - 7x - 49$$

$$\boxed{x^2 - 14x - 49}$$

Notice that by distributing you have put each expression in standard form, $ax^2 + bx + c$

How does the number in parentheses relate to the b coefficient?

The number in parentheses is half of the middle coefficient (b)

How does the number in parentheses relate to the constant, c ?

If you square the number in parentheses, you get the constant (c)

This kind of relationship always exists with a perfect square quadratic. We will use this to do a process called completing the square, which makes any expression into a perfect square quadratic.

Example 2: Find the value of c that completes the square.

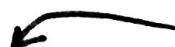
a) $x^2 - 6x + c$ $c=9$
 $-6 \div 2 = -3$
 $(-3)^2 = 9 \rightarrow (x-3)^2$

b) $x^2 + 36x + c$ $c=324$
 $36 \div 2 = 18$
 $(18)^2 = 324 \rightarrow (x+18)^2$

c) $x^2 - 20x + c$ $c=100$
 $-20 \div 2 = -10$
 $(-10)^2 = 100 \rightarrow (x-10)^2$

Example 2: Rewrite each equation by completing the square

a) $x^2 + 10x - 11 = 0$



b) $x^2 - 8x - 9 = 0$

Notice this is not a perfect square since the two properties we know are not met.

$$-8 \div 2 = -4$$

$$(-4)^2 = 16$$

c) $x^2 + 20x + 36 = 0$

d) $x^2 - 12x - 85 = 0$

O The process of completing the square makes any quadratic into a perfect square. We do this process so we can solve by taking a square root.

Steps to Complete the Square:

- 1) Move constant to other side of equation
- 2) Find the value that completes the square & add to both sides
 - Divide middle coefficient by 2 & square it
- 3) Rewrite as $(\dots)^2$

Example 4: Solve each equation by completing the square.

a) $x^2 + 8x + 15 = 0$
 $\textcircled{1} \quad x^2 + 8x = -15$

$\textcircled{2} \quad x^2 + 8x + \underline{16} = -15 + \underline{16}$
 $8 \div 2 = 4$
 $(4)^2 = 16$

$\textcircled{3} \quad (x+4)^2 = 1$

$\sqrt{(x+4)^2} = \sqrt{1}$

$x+4 = \pm 1$

$-4 \quad -4$

$x = -4 \pm 1 \quad -4+1 = \boxed{-3}$

$x = -4 \pm 1 \quad -4-1 = \boxed{-5}$

c) $x^2 - 12x - 85 = 0$
 $\textcircled{1} \quad x^2 - 12x = 85$

$\textcircled{2} \quad x^2 - 12x + \underline{36} = 85 + \underline{36}$

$-12 \div 2 = -6$

$(-6)^2 = 36$

$\textcircled{3} \quad (x-6)^2 = 121$

$\sqrt{(x-6)^2} = \sqrt{121}$

$x-6 = \pm 11$

$x = 6 \pm 11 \quad 6+11 = \boxed{17}$

$x = 6 \pm 11 \quad 6-11 = \boxed{-5}$

b) $x^2 + 18x + 56 = 0$

When we find the value that completes the square, we need to add it to both sides.

After successfully completing the square, solve by taking a square root.

d) $x^2 + 6x + 58 = 0$
 $\textcircled{1} \quad -58 \quad -58$

$x^2 + 6x + \underline{9} = -58 + \underline{9}$

$6 \div 2 = 3$

$3^2 = 9$

$(x+3)^2 = -49$

$\sqrt{(x+3)^2} = \sqrt{-49}$

$x+3 = \pm 7i$

$x = -3 \pm 7i$

$\boxed{-3+7i}$

$\boxed{-3-7i}$