4.3 Solving by Completing the Square

Today we are going to look at perfect square trinomials, meaning the trinomials that are a result of a factor being squared.

Example 1: Distribute the following expressions.

a) $(x+5)^2$

$(x+5)(x+5)$

$x^2+5x+5x+25$

$\boxed{x^2+10x+25}$

b) $(x+3)^2$

$(x+3)(x+3)$

$x^2+3x+3x+9$

$\boxed{x^2+6x+9}$

c) $(x-7)^2$

$(x-7)(x-7)$

$x^2-7x-7x+49$

$\boxed{x^2-14x+49}$

Let's look at some patterns going on here: ax^2+bx+c

1) How does the *b* (middle) coefficient relate to the number in the factor?

It is twice the number in the factor

2) How does the number in the factor relate to the constant, *c*?

You square it to get c

This kind of relationship always exists with a perfect square quadratic. We will use this to do a process called completing the square, which makes any expression into a perfect square quadratic.

Example 2: Determine if the following are perfect square trinomials. Explain your answer.

a) $x^2+10x-11=0$

$10\div 2=5$ $(x+5)^2$

$5^2=25$

No, it should be $x^2+10x+25=0$

b) $x^2-8x+64=0$

$-8\div 2=-4$ $(x-4)^2$

$(-4)^2=16$

No, it should be $x^2-8x+16=0$

Example 3: Find the value of c that completes the square.

a) x^2-6x+c

$6\div 2=-3$

$(-3)^2=\boxed{9}$

b) $x^2+36x+c$

$36\div 2=18$

$18^2=\boxed{324}$

c) $x^2-20x+c$

$-20\div 2=-10$

$(-10)^2=\boxed{100}$

Now we are going to practice a process called **completing the square.** The reason that we complete the square is so that we can solve any expression by taking a square root.

Steps to Complete the Square:

1) Move constant to other side, put blanks on both sides
2) Find the value that completes the square and add it to both sides
   - divide b by 2, then square it
3) Rewrite as $(\quad)^2$
4) Solve by taking a $\sqrt{}$

Example 4: Solve each equation by completing the square.

The easiest time to use complete the square to solve is when $a=1$ and b is even.

a) $x^2 + 8x + 15 = 0$

$8 \div 2 = 4$, $4^2 = 16$

$x^2 + 8x + 16 = -15 + 16$

$\sqrt{(x+4)^2} = \sqrt{1}$

$x + 4 = \pm 1$

$-4 \quad -4$

$x = -4 \pm 1$

$-4+1 = \boxed{-3}$

$-4-1 = \boxed{-5}$

b) $x^2 + 18x + 10 = 0$

$18 \div 2 = 9$, $9^2 = 81$

$x^2 + 18x + 81 = -10 + 81$

$\sqrt{(x+9)^2} = \sqrt{71}$

$x + 9 = \pm\sqrt{71}$

$\boxed{x = -9 \pm \sqrt{71}}$

c) $x^2 - 12x - 31 = 0$

$-12 \div 2 = -6$, $(-6)^2 = 36$

$x^2 - 12x + 36 = 31 + 36$

$\sqrt{(x-6)^2} = \sqrt{67}$

$x - 6 = \pm\sqrt{67}$

$\boxed{x = 6 \pm \sqrt{67}}$

d) $x^2 + 6x + 58 = 0$

$6 \div 2 = 3$, $3^2 = 9$

$x^2 + 6x + 9 = -58 + 9$

$\sqrt{(x+3)^2} = \sqrt{-49}$

$x + 3 = \pm 7i$

$-3 \quad -3$

$\boxed{x = -3 \pm 7i}$

When solving, when **can** you tell if you'll have rational, irrational, or imaginary solutions?

When you take the square root