4.2 Solving with Square Roots

When you are asked to solve an equation, you might want to consider solving by taking a square root. This strategy is best used when the only x that is in your equation is squared.

Important Reminder:
When you take a square root, your answer needs ±

1) Solve the following equations:

a) $\sqrt{x^2} = \sqrt{25}$

$x = \pm 5$

b) $\sqrt{x^2} = \sqrt{28}$

$x = \pm 2\sqrt{7}$

$\sqrt{28}$ → $2\sqrt{14}$ → $2\sqrt{2}\sqrt{7}$

c) $\sqrt{x^2} = \sqrt{-10}$

$x = \pm i\sqrt{10}$

$i\sqrt{10}$

2) Solve the following equations:

a) $5x^2 = 125$

$\frac{5}{5} \quad \frac{5}{5}$

$\sqrt{x^2} = \sqrt{25}$

$x = \pm 5$

b) $6x^2 = 12$

$\frac{6}{6} \quad \frac{6}{6}$

$\sqrt{x^2} = \sqrt{2}$

$x = \pm\sqrt{2}$

c) $\frac{4x^2}{4} = \frac{-4}{4}$

$\sqrt{x^2} = \sqrt{-1}$

$x = \pm i$

3) Solve the following equations:

a) $\sqrt{(x-2)^2} = \sqrt{49}$

$x - 2 = \pm 7$

$+2 \quad +2$

$x = 2 \pm 7$ → $2+7 = 9$, $2-7 = -5$

*add in front of ±

b) $\sqrt{(x+3)^2} = \sqrt{18}$

$\sqrt{18}$ → $9 \cdot 2$ → $3\sqrt{2}$

$x + 3 = \pm 3\sqrt{2}$

$-3 \quad -3$

$x = -3 \pm 3\sqrt{2}$

c) $\sqrt{(x+1)^2} = \sqrt{-4}$

$x + 1 = \pm 2i$

$-1 \quad -1$

$x = -1 \pm 2i$

4) Solve the following equations:

a) $(x-8)^2 - 5 = -1$

$(x-8)^2 = 4$

$x - 8 = \pm 2$

$x = 8 \pm 2$

$x = 10, 6$

b) $3x^2 - 10 = 152$

$3x^2 = 162$

$x^2 = 54$

$x = \pm\sqrt{54}$

$x = \pm 3\sqrt{6}$

c) $4(x+4)^2 + 5 = 21$

$4(x+4)^2 = 16$

$(x+4)^2 = 4$

$x + 4 = \pm 2$

$x = -4 \pm 2$

$x = -2, -6$

Big idea of solving using a square root: Get the squared term by itself, then take a square root

4) Solve each equation.

a) $(x+10)^2 - 225 = 0$

$+225 \quad +225$

$\sqrt{(x+10)^2} = \sqrt{225}$

$x+10 = \pm 15$

$-10 \quad -10$

$x = -10 \pm 15$ → $-10+15 = 5$, $-10-15 = -25$

b) $(x-1)^2 + 6 = 26$

$-6 \quad -6$

$\sqrt{(x-1)^2} = \sqrt{20}$

$\sqrt{20}$ → $4 \cdot 5$ → $2 \cdot 2$

$x-1 = \pm 2\sqrt{5}$

$+1 \quad +1$

$x = 1 \pm 2\sqrt{5}$

c) $9x^2 + 80 = 81$

$-80 \quad -80$

$\frac{9x^2}{9} = \frac{1}{9}$

$\sqrt{x^2} = \sqrt{\frac{1}{9}} = \frac{\sqrt{1}}{\sqrt{9}}$

$x = \pm \frac{1}{3}$

*To take the $\sqrt{}$ of a fraction, take the $\sqrt{}$ of the top over the $\sqrt{}$ of the bottom

d) $5(x-7)^2 - 25 = 10$

$5(x-7)^2 = 35$

$\sqrt{(x-7)^2} = \sqrt{7}$

$x-7 = \pm\sqrt{7}$

$x = 7 \pm \sqrt{7}$

e) $x^2 - 3 = 37$

$+3 \quad +3$

$\sqrt{x^2} = \sqrt{40}$

$\sqrt{40}$ → $4 \cdot 10$ → $2 \cdot 2 \cdot 2 \cdot 5$

$x = \pm 2\sqrt{10}$

f) $4(x+6)^2 - 5 = 12$

$4(x+6)^2 = 17$

$\sqrt{(x+6)^2} = \sqrt{\frac{17}{4}}$

$x+6 = \pm \frac{\sqrt{17}}{2}$

$x = -6 \pm \frac{\sqrt{17}}{2}$

As you go through these problems, the most important thing you can do is ask yourself, "Which term is being squared?" Once you have identified that, your goal is to get that term by itself.

Fundamental Theorem of Algebra: The degree (highest exponent) of an equation tells you how many solutions it has.

ex: $x^3 + 2x^2 - 3 = 0$ has 3 solutions

$x^2 + 7x + 10 = 0$ has 2 solutions