4.1 Solving by Factoring

1) Factor each expression:

a) $p^2 - 10p + 9$ $9p^2$

$\boxed{(p-9)(p-1)}$ $-9p$ $-1p$

b) $2b^2 - 12b - 54$ $-27b^2$

$2(b^2 - 6b - 27)$ $-9b$ $3b$

$\boxed{2(b-9)(b+3)}$

c) $n^2 - \sqrt{81}$ Missing middle term

$(n+9)(n-9)$

d) $2x^2 - 17x - 9$ $-18x^2$

$2x^2 - 18x + 1x - 9$ $-18x$ $1x$

$2x(x-9) + 1(x-9)$

$\boxed{(2x+1)(x-9)}$

To introduce solving by factoring, we need to talk about something called the **zero product property.** Here's what it looks like:

a. Solve $3x = 0$

What does x equal? How do you know?

$x = 0$ since anything times 0 is 0

b. Solve $3(x + 1) = 0$

$x + 1 = 0$
$-1 \quad -1$
$x = -1$

The **zero product property** says that if two things multiply to be 0, one of these things must be 0.

What is a quick way we can see the solution to this equation without taking all the steps to solve?

Take the opposite sign of the number in the factor.

If we can break down an equation into its factors, then solving becomes a lot faster using the method you just discovered.

2) Solve each of the following:

a) $p^2 - 10p + 9 = 0$ $9p^2$

$(p-9)(p-1) = 0$ $-9p$ $-1p$

$\boxed{p = 9, 1}$

We are finding the values that would make each factor 0 when we plug it in.

ex. $(p-9)(p-1) = 0$

$\boxed{p=9}$ $9-9=0$ $1-1=0$ $\boxed{p=1}$

b) $n^2 - 81 = 0$ Use difference of squares to factor.

$(n+9)(n-9) = 0$

$\boxed{n = -9, 9}$

The big idea behind solving by factoring:

Factor & take the opposite of the number in the factor.

Before you begin solving, one side must equal 0 and the equation must be in standard form.

3) Solve each equation.

a) $k^2 + 9k + 14 = 0$ $\quad 14k^2$

$(k+7)(k+2)=0$ $\quad 7k \ 2k$

$\boxed{k=-7,-2}$

b) $-x^2 - 4x - 4 = 0$

$-(x^2+4x+4)=0$ $\quad 4x^2$, $2x \ 2x$

$-(x+2)(x+2)=0$

$\boxed{x=-2,-2}$

(Tip #1)

Tip #1
A GCF out front does not affect your solutions.

c) $6k^2 + 7k = -2$

$+2 \quad +2$

$12k^2$, $4k \ 3k$

$6k^2+7k+2=0$

$6k^2+4k+3k+2=0$

$2k(3k+2)+1(3k+2)=0$

$(2k+1)(3k+2)=0$

$\boxed{k=-\frac{1}{2}, -\frac{2}{3}}$ (Tip #2)

d) $x^2 - 22 = 3$

$-3 \quad -3$

$x^2-25=0$

$(x+5)(x-5)=0$

$\boxed{x=-5, 5}$

Tip #2
$(x-1)(3x-4)=0$
$x=1$ $\quad \boxed{x=\frac{4}{3}}$
Take opposite & divide by coefficient

e) $n^2 - 7n = 0$

$n(n-7)=0$ GCF

$\boxed{n=0,7}$

(Tip #3)

f) $8x^2 + 2x - 6 = -5$

$+5 \quad +5$

$8x^2+2x-1=0$ $\quad -8x^2$, $4x \ -2x$

$8x^2+4x-2x-1=0$

$4x(2x+1)-1(2x+1)=0$

$(4x-1)(2x+1)=0$

$\boxed{x=\frac{1}{4}, -\frac{1}{2}}$ (Tip #2)

Tip #3
A variable by itself out front gives a solution of $x=0$
ex: $x(x-5)=0$ $\quad \boxed{x=0, x=5}$

g) $2x^2 - 17x - 9 = 0$ $\quad -18x^2$, $-18x \ 1x$

$2x^2-18x+1x-9=0$

$2x(x-9)+1(x-9)=0$

$(2x+1)(x-9)=0$

$\boxed{x=-\frac{1}{2}, 9}$

h) $4a^2 + 121 = 0$

Difference of squares

$(2a+11i)(2a-11i)=0$

$\boxed{a=-\frac{11i}{2}, \frac{11i}{2}}$