2.2: Simplifying Radicals

A **RADICAL** is an expression that involves a square root, cube root, etc. Today we will simplify radicals in the real number system and in the complex number system (resulting in imaginary numbers).

We are looking for groups of numbers that are multiplied by themselves (since that's what roots ask for).

THE REAL NUMBER SYSTEM

How do we simplify a radical:

1. Make a factor tree
2. Find groups of the same number.
3. Any groups come out. Non-groups stay in the radical.
4. Multiply everything outside the radical.
5. Multiply everything inside the radical.

a) $\sqrt{392}$

392 → 2, 196; 196 → 2, 98; 98 → 2, 49; 49 → 7, 7

$2 \cdot 7\sqrt{2}$

$\boxed{14\sqrt{2}}$

Anything without a pair stays inside the radical.

b) $\sqrt[3]{24}$ — This means we circle groups of 3

24 → 2, 12; 12 → 2, 6; 6 → 2, 3

$\boxed{2\sqrt[3]{3}}$

c) $\sqrt{63}$

63 → 9, 7; 9 → 3, 3

$\boxed{3\sqrt{7}}$

d) $2\sqrt{112}$

A number out front gets multiplied with numbers that come out

112 → 2, 56; 56 → 8, 7; 8 → 4, 2; 4 → 2, 2

$2 \cdot 2 \cdot 2\sqrt{7}$

$\boxed{8\sqrt{7}}$

e) $6\sqrt{225}$

225 → 15, 15

$6 \cdot 15 = \boxed{90}$

Variables change things a bit because a variable can be positive or negative. It is sometimes necessary to restrict your answer to ONLY positive numbers. To do this, you use absolute values in your solution.

You need absolute values in your solution when:
1) Real
2) $\sqrt{\ }$ or $\sqrt[4]{\ }$ (or any even root)
3) Odd amount of variable comes out

f) $\sqrt{128x^4}$

128 → 2, 64; 64 → 8, 8; x x, x x

$\boxed{8x^2\sqrt{2}}$

Don't need abs. value since an even amount of x's came out

g) $\sqrt[3]{72x^4}$ — Don't need absolute value since not even root

72 → 2, 36; 36 → 2, 18; 18 → 2, 9; 9 → 3, 3; x x x, x

$\boxed{2x\sqrt[3]{9x}}$

h) $2\sqrt{12x^3}$

12 → 6, 2; 6 → 3, 2; x x, x

$2 \cdot 2x\sqrt{3x}$

$4x\sqrt{3x}$

$\boxed{4|x|\sqrt{3x}}$

We need abs. value since it is a square root and only one x (odd amount) came out

i) $3\sqrt{175x^3y^2}$

175 → 25, 7; 25 → 5, 5; x x, x; y y

$3 \cdot 5xy\sqrt{7x}$

$\boxed{15|xy|\sqrt{7x}}$

j) $6\sqrt{24x^4y^2}$

24 → 6, 4; 6 → 3, 2; 4 → 2, 2; x x, x x; y y

$6 \cdot 2x^2y\sqrt{6}$

$\boxed{12x^2|y|\sqrt{6}}$

Only y needs abs value since even amount of x's came out

k) $\sqrt[3]{64x^6}$

64 → 2, 32; 32 → 2, 16; 16 → 4, 4; 4 → 2, 2; 4 → 2, 2; x x x, x x x

$2 \cdot 2x^2$

$\boxed{4x^2}$

THE COMPLEX NUMBER SYSTEM

If the numbers are imaginary, then we do not need absolute value signs

The complex number system assumes that there WILL be negatives inside of the radical. Because of this – absolute value signs are NOT necessary when solving the problem!

a) $\sqrt{-20}$

$i\sqrt{20}$

4 5

2 2

$2i\sqrt{5}$

b) $\sqrt{-80}$

$i\sqrt{80}$

20 4

10 2 2 2

5 2

$2\cdot 2i\sqrt{5}$

$4i\sqrt{5}$

c) $\sqrt{-120}$

$i\sqrt{120}$

40 3

2 20

2 10

2 5

$2i\sqrt{2\cdot 5\cdot 3}$

$2i\sqrt{30}$

d) $\sqrt{-16}$

$i\sqrt{16}$

$4i$

e) $\sqrt{-1000}$

f) $\sqrt{-50}$

g) $\sqrt{-25x^2y^4}$

$i\sqrt{25x^2y^4}$

5 5 x x y y y y

$5ixy^2$

h) $\sqrt{-60xy^3}$

$i\sqrt{60xy^3}$

6 10 x y y y

2 3 2 5

$2iy\sqrt{3\cdot 5x}$

$2iy\sqrt{15x}$

i) $\sqrt{-27x^{10}y^7}$

$i\sqrt{27x^{10}y^7}$

9 3

3 3

x x x x x x x x x x y y y y y y y

$3ix^5y^3\sqrt{3y}$