2.2 Simplifying Radicals

A **radical** is an expression that involves a square root, cube root, etc. Not all numbers come out with a perfect root, but when we break down a radical, you may find that there are some numbers that can simplify out of the radical.

1) Simplify each radical.

Steps to Simplifying Radicals

1) Make a prime factor tree
2) Circle groups of the kind of root you take ex: $\sqrt{\ }$ - groups of 2 $\sqrt[3]{\ }$ - groups of 3
3) Circled groups come out, everything else stays in
   - multiply everything outside & everything inside

a) $\sqrt{63}$

square root is always groups of 2

3 $\cancel{21}$

3 7

$\boxed{3\sqrt{7}}$

b) $\sqrt{392}$

Even numbers are always divisible by 2

2 $\cancel{196}$

2 $\cancel{98}$

2 $\cancel{49}$

7 7

$2 \cdot 7\sqrt{2}$

$\boxed{14\sqrt{2}}$

c) $3\sqrt{112}$

2 $\cancel{56}$

2 $\cancel{28}$

2 $\cancel{14}$

2 7

$3 \cdot 2 \cdot 2\sqrt{7}$

$\boxed{12\sqrt{7}}$

d) $\sqrt[3]{56}$ ← Circle groups of 3

2 $\cancel{28}$

2 $\cancel{14}$

2 7

$\boxed{2\sqrt[3]{7}}$

*Make sure to keep small 3 in answer

f) $\sqrt[3]{-132}$

$+ \cdot + \cdot + = +$

$- \cdot - \cdot - = -$

$+$

$-\sqrt[3]{132}$

2 66

2 33

Odd kinds of roots can have negative answers

$\boxed{-\sqrt[3]{132}}$

If nothing gets circled, nothing comes out

g) $4\sqrt[3]{500}$

5 $\cancel{100}$

5 $\cancel{20}$

5 $\cancel{4}$

2 2

$4 \cdot 5\sqrt[3]{2 \cdot 2}$

$\boxed{20\sqrt[3]{4}}$

h) $\sqrt[4]{405}$

Anything that ends in 0 or 5 is always divisible by 5

5 $\cancel{81}$

$\cancel{9}$ $\cancel{9}$

3 3 3 3

$\boxed{3\sqrt[4]{5}}$

You will also be asked to simplify radicals with imaginary roots. To do this, rip the i out and take the square root as normal.

2) Simplify each radical.

a) $\sqrt{-20} = i\sqrt{20}$

5 ~~4~~

2 2

$\boxed{2i\sqrt{5}}$

b) $\sqrt{-80} = i\sqrt{80}$

~~8~~ ~~10~~

~~4~~ 2 2 5

2 2

$\boxed{4i\sqrt{5}}$

c) $\sqrt{-120} = i\sqrt{120}$

~~12~~ ~~10~~

~~6~~ 2 2 5

2 3

$2i\sqrt{2 \cdot 3 \cdot 5} = \boxed{2i\sqrt{30}}$

d) $\sqrt{-16} = i\sqrt{16}$

4 4

$\boxed{4i}$

If everything gets circled, the root goes away

e) $\sqrt{-1000} = i\sqrt{1000}$

~~10~~ ~~100~~

5 2 10 10

$\boxed{10i\sqrt{10}}$

f) $\sqrt{-50} = i\sqrt{50}$

5 ~~10~~

5 2

$\boxed{5i\sqrt{2}}$

*i only happens with a square root of a negative

If you encounter radicals with variables, you will still break it down like normal; however, the variables that come out of the radical may need absolute value signs around them.

When you are asked to find a root, you are being asked for the **principal root**, meaning the positive outcome of the square root. Because the variable might be negative, we need absolute value signs to assure that we are answering with the positive root.

Variables outside the radical need absolute value when:
1) Real
2) Even kind of root $\sqrt{\ }$, $\sqrt[4]{\ }$
3) Odd amount of variable comes out

b) $\sqrt{24x^4y^2}$

~~6~~ ~~4~~ x x x x y y

3 2 2 2

Real ✓
Even root ✓
2 x's – no abs value
1 y – abs value

$\boxed{2x^2|y|\sqrt{6}}$

c) $\sqrt{12x^3}$

2 6 x x x

2 3

Real ✓
Even root ✓
1 x – odd, abs value

$\boxed{2|x|\sqrt{3x}}$

d) $\sqrt{128x^4}$

2 64 x x x x

8 8

Real ✓
Even root ✓
2 x's – even, no abs value

$\boxed{8x^2\sqrt{2}}$

e) $2\sqrt{175x^3y^2}$

5 35 x x x y y

5 7

Real ✓
Even root ✓
1 x – abs value
1 y – abs value

$2 \cdot 5\, x y \sqrt{7x}$

$\boxed{10|xy|\sqrt{7x}}$

f) $\sqrt[3]{-72x^4} = -\sqrt[3]{72x^4}$

2 36 x x x x

2 ~~18~~

2 ~~9~~

3 3

Real ✓
Even root ✗
No abs value

$\boxed{-2x\sqrt[3]{9x}}$