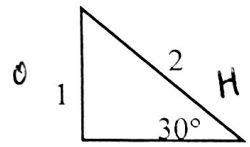
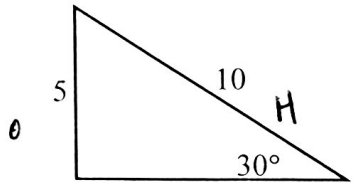
11.3 Missing Angles

Let's explore a little bit with trig ratios:

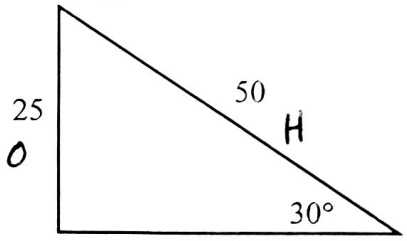
a. sin30= $\frac{1}{2}$



b. sin30= $\frac{5}{10} = \frac{1}{2}$



c. sin30= $\frac{25}{50} = \frac{1}{2}$



What do you notice about each value of sine? All reduce to the same thing

Any triangles with the same angle measures are similar (like we discussed in unit 10). Because they are similar, we know that all of the side lengths are proportional. This is why any trig ratio of the same angle will always reduce to the same value.

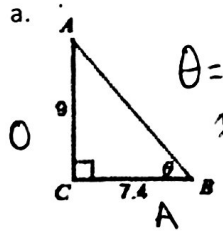
~~Translation: Sine, cosine, and tangent of any angle are fixed values.~~

Why do we care? Well, since we know trig ratios are a fixed value, we can now match the value any trig ratio to the angle that it came from. We do this with something called **inverse trig**.

$\sin^{-1}(\frac{\text{opposite}}{\text{hypotenuse}}) = \theta$	$\cos^{-1}(\frac{\text{adjacent}}{\text{hypotenuse}}) = \theta$	$\tan^{-1}(\frac{\text{opposite}}{\text{adjacent}}) = \theta$

1) Find the indicated angle. Round your answer to the nearest hundredth.

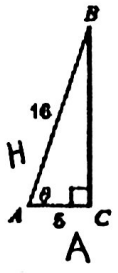
a.



$\theta = \tan^{-1}(\frac{9}{7.4})$

$\approx 50.57°$

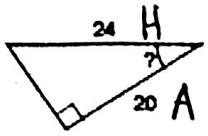
b.



$\theta = \cos^{-1}(\frac{5}{16})$

$\approx 71.79°$

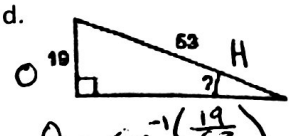
c.



$\theta = \cos^{-1}(\frac{20}{24})$

$\approx 33.56°$

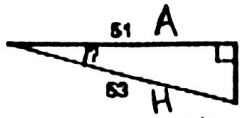
d.



$\theta = \sin^{-1}(\frac{19}{53})$

$\approx 21.01°$

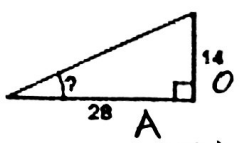
e.



$\theta = \cos^{-1}(\frac{51}{53})$

$\approx 15.79°$

f.

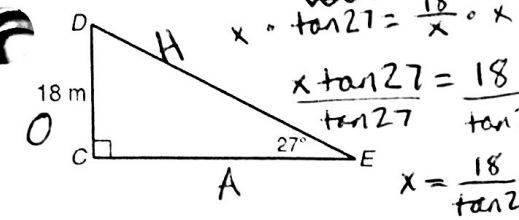


$\theta = \tan^{-1}(\frac{14}{28})$

$\approx 26.57°$

Solving a triangle means that we need to mind the measure of every side and angle.

2) Solve each triangle.



CE

$x \cdot \tan 27 = \frac{18}{x} \cdot x$

$\frac{x \tan 27}{\tan 27} = \frac{18}{\tan 27}$

$x = \frac{18}{\tan 27}$

≈ 35.33

CD= 18 m m∠C= 90°

DE= 39.65 m m∠D= 180−90−27= 63°

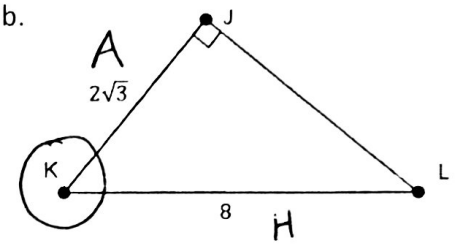
CE= 35.33 m m∠E= 27°

DE

$\sin 27 = \frac{18}{x}$

$x = \frac{18}{\sin 27} \approx 39.65$

b.



KJ= $2\sqrt{3}$ m∠K= $\theta = \cos^{-1}\left(\frac{2\sqrt{3}}{8}\right) \approx 64.34°$

JL= $2\sqrt{13}$ m∠J= 90°

KL= 8 m∠L= 180−90−64.34= 25.66°

$(2\sqrt{3})^2 + b^2 = 8^2$

$12 + b^2 = 64$

$b^2 = 52$

$b = \sqrt{52}$

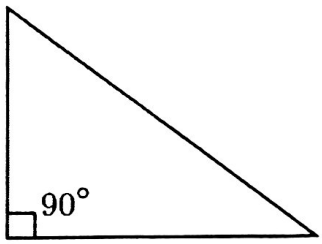
$2 \cdot 26$

$2 \cdot 13$

$b = 2\sqrt{13}$

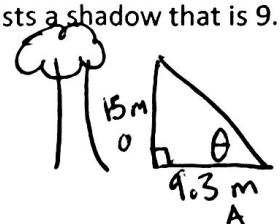
Word Problems

When you are asked about an **angle of elevation** or an **angle of depression**, it Is referring to the angle in the bottom corner of the triangle. There is a difference between angle of elevation and angle of depression, but for our class, we are going to treat them as if they are the same thing (because they're congruent anyway).



Remember, the best way to tackle a word problem is to draw it.

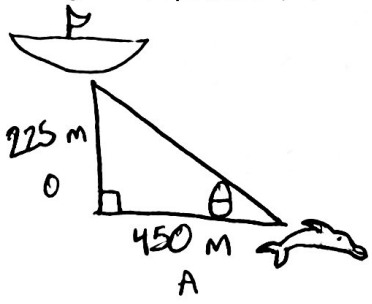
3) The height of a tree is 15 meters. To the nearest degree, what is the angle of elevation of the sun when the tree casts a shadow that is 9.3 meters long on ground level?



$\theta = \tan^{-1}\left(\frac{15}{9.3}\right)$

$\approx 58.20°$

4) A sonar operator on an anchored cruiser detects a pod of dolphins feeding at a depth of about 255 meters directly below his ship. If the cruiser travels 450 meters west and the dolphins remain at the same depth to feed, what is the angle of depression, x, from the cruiser to the pod?



$\theta = \tan^{-1}\left(\frac{225}{450}\right)$

$\approx 26.57°$