

1.4 Simplifying Radicals

Reducing radicals:

- 1) Make a factor tree
- 2) Circle groups of the index
- 3) Circled groups come out, everything else stays in
 - multiply everything outside radical & everything inside radical

1) Simplify.

a) $\sqrt{63}$

Factor tree: 63 → 9 × 7 (9 is circled)

$= 3\sqrt{7}$

b) $\sqrt{392}$

Factor tree: 392 → 2 × 196 → 2 × 98 → 2 × 49 → 2 × 7 × 7 (two 7s are circled)

$= 2 \cdot 7 \sqrt{2}$
 $= 14\sqrt{2}$

c) $3\sqrt{112}$

Factor tree: 112 → 2 × 56 → 2 × 28 → 2 × 14 → 2 × 7 (two 2s are circled)

$= 3 \cdot 2 \cdot 2 \sqrt{7}$
 $= 12\sqrt{7}$

**don't forget the 3 that was already out front*

d) $\sqrt[3]{108}$

Factor tree: 108 → 2 × 54 → 2 × 27 → 3 × 9 → 3 × 3 × 3 (three 3s are circled)

$= 3\sqrt[3]{2 \cdot 2}$
 $= 3\sqrt[3]{4}$

When you are taking a root of an even index, by definition you are taking the positive root (this is called the principal root). Because of this, sometimes when we reduce roots with variables we need absolute value signs.

Absolute value signs:

- 1) Real number
- 2) Even index
- 3) Odd amount of variable comes out

2) Simplify. Use absolute value signs when necessary.

*all 3 conditions must be met to need absolute value

a) $\sqrt{12x^3}$

Factor tree: 12 → 2 × 6 → 2 × 3 (two 2s are circled)

Real ✓
 Even index ✓
 Odd amount of variable ✓

$= 2|x|\sqrt{3}$

b) $\sqrt{128x^4}$

Factor tree: 128 → 2 × 64 → 2 × 32 → 2 × 16 → 2 × 8 → 2 × 4 → 2 × 2 (two 8s are circled)

Real ✓
 Even index ✓
 Odd amount of variable ✓

$= 8x^2\sqrt{2}$

c) $\sqrt[3]{72x^4}$

Factor tree: 72 → 2 × 36 → 2 × 18 → 2 × 9 → 3 × 3 (two 2s and one 3 are circled)

Real ✓
 Even index x

$= 2x\sqrt[3]{3 \cdot 3x}$
 $= 2x\sqrt[3]{9x}$

d) $6\sqrt{24x^4y^2}$

Factor tree: 24 → 2 × 12 → 2 × 6 → 2 × 3 (two 2s are circled)

Real ✓
 Even index ✓
 Odd amount of variable ✓

$= 6 \cdot 2x^2|y|\sqrt{6} = 12x^2|y|\sqrt{6}$

- adding/subtracting radicals

- reduce all radicals first
- combine like terms - add coefficients of like radicals

3) Simplify.

$$\begin{aligned} \text{a) } \sqrt{8} + 4\sqrt{2} &= 2\sqrt{2} + 4\sqrt{2} \\ &= \boxed{6\sqrt{2}} \end{aligned}$$

$$\begin{aligned} \text{b) } -2\sqrt{5} + 6\sqrt{3} - 2\sqrt{20} - \sqrt{12} \\ &= -2\sqrt{5} + 6\sqrt{3} - 2 \cdot 2\sqrt{5} - 2\sqrt{3} \\ &= -2\sqrt{5} + 6\sqrt{3} - 4\sqrt{5} - 2\sqrt{3} = \boxed{-6\sqrt{5} + 4\sqrt{3}} \end{aligned}$$

- multiplying radicals

- when roots have same index, you can multiply the insides and combine under the same root

4) Simplify.

$$\begin{aligned} \text{a) } \sqrt{10} \cdot \sqrt{15} &= \sqrt{150} \\ &= \sqrt{25 \cdot 6} \\ &= \boxed{5\sqrt{6}} \end{aligned}$$

$$\begin{aligned} \text{b) } \sqrt[3]{5x^2} \cdot \sqrt[3]{25x} &= \sqrt[3]{125x^3} \\ &= \boxed{5x} \end{aligned}$$

- dividing radicals

- rationalize denominator - no roots in denominator
- if there is no addition/subtraction in denominator, multiply top & bottom by root of denominator

5) Simplify.

$$\text{a) } \frac{5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{5\sqrt{2}}{2}}$$

$$\text{b) } \frac{2}{6\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{6 \cdot 3} = \boxed{\frac{\sqrt{3}}{9}}$$

- if there is addition/subtraction in denominator, we will use what we call the conjugate
 - conjugate - when you switch the sign in the middle of a binomial (two terms)
 - conjugates come from the \pm option when solving square roots

$$\text{c) } \frac{3}{4+\sqrt{2}} \cdot \frac{(4-\sqrt{2})}{(4-\sqrt{2})} = \boxed{\frac{12-3\sqrt{2}}{14}}$$

$$\begin{aligned} (4+\sqrt{2})(4-\sqrt{2}) \\ 16 - 4\sqrt{2} + 4\sqrt{2} - 2 \\ 16 - 2 = 14 \end{aligned}$$