1.3 Properties of Exponents (Rational)

- If you have a rational number (fraction) as an exponent, it means that you are taking a root.

$\sqrt[\square]{x^{\square}}$ ← power of base vs. $x^{\frac{\square}{\square}}$ ← power of base, index

index: power of the root you take

1) Switch between radicals & rational exponents.

a) $y^{\frac{2}{3}} = \boxed{\sqrt[3]{y^2}}$

b) $3^{\frac{1}{2}} = \boxed{\sqrt{3}}$

c) $2x^{\frac{2}{5}} = \boxed{2\sqrt[5]{x^2}}$

d) $\sqrt[3]{(6x)^4} = \boxed{(6x)^{\frac{4}{3}}}$

e) $\sqrt{5} = \boxed{5^{\frac{1}{2}}}$

f) $\sqrt[5]{x^4} = \boxed{x^{\frac{4}{5}}}$

- All of the exponent rules that we practiced with integers also apply to rational exponents.

2) Simplify. Your answer should contain only positive exponents.

a) $(m^{\frac{1}{2}})^3 = \boxed{m^{\frac{3}{2}}}$

b) $(m^{\frac{1}{2}} \cdot m^{\frac{2}{5}})^4$

$\frac{1}{2} + \frac{2}{5} = \frac{9}{10}$

$(m^{\frac{9}{10}})^4 = m^{\frac{36}{10}} = \boxed{m^{\frac{18}{5}}}$

c) $\frac{x^{\frac{3}{5}}}{x^{\frac{1}{2}}} = \boxed{x^{\frac{1}{10}}}$ $\qquad \frac{3}{5} - \frac{1}{2} = \frac{1}{10}$

d) $\frac{(4x^{\frac{1}{4}})^2}{x^{\frac{1}{2}}} = \frac{16x^{\frac{2}{4}}}{x^{\frac{1}{2}}} = \frac{16x^{\frac{1}{2}}}{x^{\frac{1}{2}}} = \boxed{16}$

- It is considered improper to a have a root in a denominator of a fraction. This means we have to rationalize the denominator. To do this, we multiply top and bottom by the power that will bring the bottom exponent to the next whole number.

ex: $\underbrace{\frac{1}{x^{\frac{3}{5}}} \cdot \frac{x^{\frac{2}{5}}}{x^{\frac{2}{5}}}}_{\text{add to get } 1} = \boxed{\frac{x^{\frac{2}{5}}}{x}}$

ex: $\frac{3}{x^{\frac{9}{8}}} \cdot \frac{x^{\frac{7}{8}}}{x^{\frac{7}{8}}} = \boxed{\frac{3x^{\frac{7}{8}}}{x^2}}$