Unit 1.1: Basic Exponent Properties

exponent a number that tells us how many time a quantity is multiplied by itself. Another word for exponent is **power**. The quantity that is being multiplied by itself is called the **base**.

Ex: $5^3 = 5 \cdot 5 \cdot 5$ $7^8 = 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$ $21^4 = 21 \cdot 21 \cdot 21 \cdot 21$

Using this information, see if you can figure out some shortcuts or rules for simplifying exponents. Be sure to show your work to help you.

Product of Powers

Simplify the following exponents:

$6^3 \cdot 6^5 = 6^8$

$6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$

$4^7 \cdot 4^2 = 4^9$

$12^4 \cdot 12^4 = 12^8$

$12 \cdot 12 \cdot 12 \cdot 12 \cdot 12 \cdot 12 \cdot 12 \cdot 12$

Add exponents

Quotient of Powers

Simplify the following exponents:

$\frac{2^7}{2^4} = 2^3$

$\frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2}$

$\frac{8^5}{8^2} = 8^3$

$\frac{8 \cdot 8 \cdot 8 \cdot 8 \cdot 8}{8 \cdot 8}$

$\frac{10^9}{10^4} = 10^5$

Subtract exponents

Power of Powers

Simplify the following exponents:

$(2^3)^3 = 2^9$

$2^3 \cdot 2^3 \cdot 2^3$

$(5^2)^4 = 5^8$

$5^2 \cdot 5^2 \cdot 5^2 \cdot 5^2$

$(8^4)^3 = 8^{12}$

Multiply exponents

Notice that to simplify the exponents, the two expressions **must** have the same base. If you ha(oefficients in front of your exponent-base pair, multiply them as normal.

Must have same bases to simplify exponents

Property	Rule	Example
Zero Property	Anything with exponent 0 is 1	• $a^0 = 1$ • $12^0 = 1$
Negative Exponent Property	Flips its spot in a fraction * Exponent only applies to what it is directly attached to	• $\left(\frac{1}{a}\right)^{-1} = \frac{a}{1} = a$ • $(2a)^{-2} = \frac{1}{(2a)^2} = \boxed{\frac{1}{4a^2}}$ • $2a^{-3} = \frac{2}{a^3}$
Product of Powers Property	Add exponents	• $a^4 \cdot a^3 = a^7$ • $5a^2 \cdot 2a^9 = 10a^{11}$
Quotient of Powers Property	Subtract exponents	• $\frac{a^7}{a^2} = a^5$ • $\frac{6a^{10}}{2a^{-1}} = \boxed{3a^{11}}$ (10−(−1))
Power of a Power Property	Multiply exponents * Exponent applies to everything in parentheses	• $(a^3)^2 = a^6$ • $(2x^2)^5 = 2^5 x^{10} = \boxed{32x^{10}}$

1) Simplify. Your answer should only contain positive exponents.

a. $(2b^4)^0 = \boxed{1}$

b. $(2b^0)^4 = \boxed{2^4} = \boxed{16}$

c. $(2m^{-2} \cdot 3m)^2$

Order of operations: parentheses first

$= (6m^{-2} \cdot m)^2$

$= (6m^{-1})^2$

$= 6^2 m^{-2} = \boxed{\frac{36}{m^2}}$

d. $\frac{(b^4)^2}{b^5} = \frac{b^8}{b^5} = \boxed{b^3}$

e. $\frac{5a^7}{4a^4} = \boxed{\frac{5a^3}{4}}$

f. $(3x^2)^{-3}$

$= 3^{-3} x^{-6}$

$= \frac{1}{3^3 x^6} = \boxed{\frac{1}{27x^6}}$

g. $\frac{6y^4 \cdot 3y^2}{2y^3} = \frac{18y^4 \cdot y^2}{2y^3}$

$= \frac{18y^6}{2y^3} = \boxed{9y^3}$

h. $\frac{6x^{-2}}{2x^4} = \frac{3x^{-2}}{x^4}$

$= 3x^{-6} = \boxed{\frac{3}{x^6}}$

i. $\frac{(3m^3)^2}{2m^{-4}} = \frac{3^2 m^6}{2m^{-4}} = \frac{9m^6}{2m^{-4}}$

$= \boxed{\frac{9m^{10}}{2}}$

* Deal with negative exponents last, tackle what is easiest first

Sometimes we see an exponent in the form of a fraction. This means that we can rewrite it into a **radical expression**. The denominator of the fraction signifies what kind of root you are taking. The numerator signifies the exponent of the base inside the root.

Ex: $3^{\frac{1}{2}} = \sqrt{3}$ $\quad m^{\frac{1}{5}} = \sqrt[5]{m}$ $\quad y^{\frac{2}{3}} = \sqrt[3]{y^2}$ $\quad 2x^{\frac{2}{5}} = 2\sqrt[5]{x^2}$

2) Convert each fractional exponent to a radical expression or vice versa.

a. $x^{\frac{4}{5}} = \sqrt[5]{x^4}$ b. $3a^{\frac{1}{3}} = 3\sqrt[3]{a}$ c. $(3a)^{\frac{2}{3}} = \sqrt[3]{(3a)^2}$

d. $\sqrt{5} = 5^{\frac{1}{2}}$ e. ~~$\sqrt[4]{2x^5}$~~ f. $\sqrt[3]{(6x)^4} = (6x)^{\frac{4}{3}}$

Let's remember some fraction rules before we apply it to exponents:

Add or subtract	Criss cross smiley face
Multiply	Multiply straight across
Divide	

3) Simplify. Your answer should contain only positive exponents.

a. $(m^{\frac{1}{2}})^3$

$= m^{\frac{1}{2}\cdot\frac{3}{1}}$

$= \boxed{m^{\frac{3}{2}}}$

b. $(m^{\frac{1}{2}} \cdot m^{\frac{2}{5}})^4$

$\frac{1}{2} + \frac{2}{5}$ ..... $= (m^{\frac{1}{2}+\frac{2}{5}})^4$

$= \frac{5+4}{10}$ $= (m^{\frac{9}{10}})^4$

$= \frac{9}{10}$ $= m^{\frac{9}{10}\cdot\frac{4}{1}} = \boxed{m^{\frac{36}{10}}}$

c. $\frac{x^{\frac{3}{5}}}{x^{\frac{1}{2}}} = x^{\frac{3}{5}-\frac{1}{2}}$

$= \boxed{x^{\frac{1}{10}}}$

$\frac{3}{5} \times \frac{1}{2}$

$= \frac{6-5}{10} = \frac{1}{10}$

d. $\frac{3x^{\frac{1}{4}}}{x^{\frac{1}{2}}} = 3x^{\frac{1}{4}-\frac{1}{2}}$

$= 3x^{-\frac{1}{4}} = \boxed{\frac{3}{x^{\frac{1}{4}}}}$

$\frac{1}{4} \times \frac{1}{2}$

$= \frac{2-4}{8} = \frac{-2}{8} = -\frac{1}{4}$

e. $(2y^{\frac{1}{2}} \cdot 6y^{\frac{2}{5}})^4$

$= (12y^{\frac{1}{2}+\frac{2}{5}})^4$ $\quad \frac{1}{2}+\frac{2}{5} = \frac{9}{10}$

$= (12y^{\frac{9}{10}})^4$

$= 12^4 y^{\frac{9}{10}\cdot\frac{4}{1}}$

$= \boxed{12^4 y^{\frac{36}{10}}}$

f. $\frac{(4x^{\frac{1}{4}})^2}{x^{\frac{1}{2}}} = \frac{4^2 x^{\frac{1}{4}\cdot\frac{2}{1}}}{x^{\frac{1}{2}}} = \frac{16x^{\frac{2}{4}}}{x^{\frac{1}{2}}}$

$= \frac{16x^{\frac{1}{2}}}{x^{\frac{1}{2}}} = 16x^{\frac{1}{2}-\frac{1}{2}}$

$= 16x^0 = \boxed{16}$