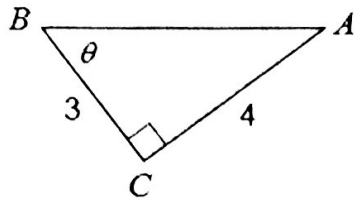
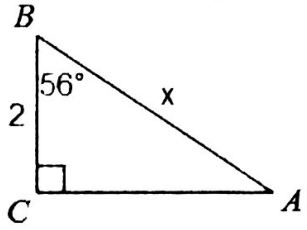
Unit 9.4 Area, Law of Sines, and Law of Cosines

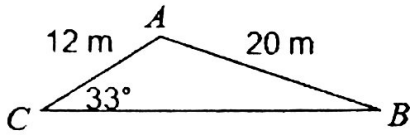
Review. Find each missing part of the triangle:

a)



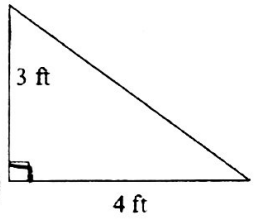
b) SOHCAHTOA





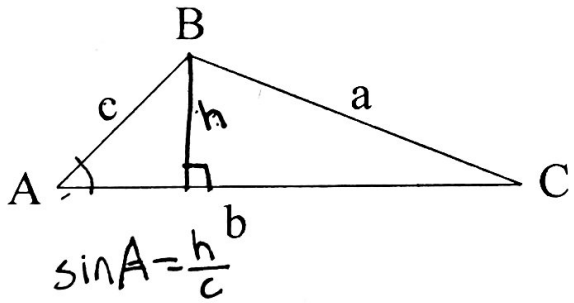
You can only use basic trigonometry when you are dealing with right triangles. What would you do if you didn't have a right triangle? Our goal for today is to be able to use trigonometry to find the area and missing sides of non-right triangles.

Review: Find the area of the triangle:



The area formula: $\frac{bh}{2}$ / $\frac{1}{2}bh$ only works when the height is perpendicular to the base. What do you do if you don't know the height?

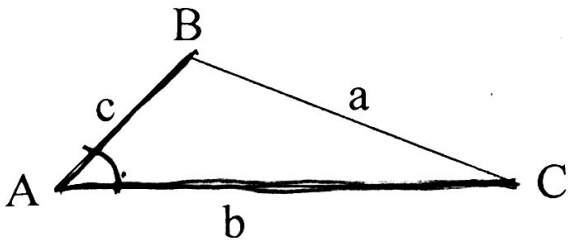
Consider the following triangle:



$\sin A = \frac{h}{c}$

Given angle A. Is it possible to find the height of the triangle?

What would the formula be for the area of the



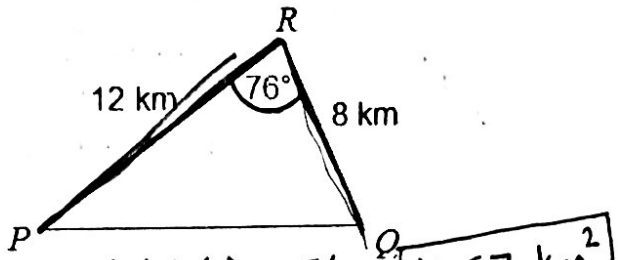
$\text{Area} = \frac{1}{2}bc\sin A$ $= \frac{1}{2}bc\frac{h}{c} = \frac{1}{2}bh$

Where A is the included angle of 2 consecutive sides b, c.

Need an angle & the two sides that make it

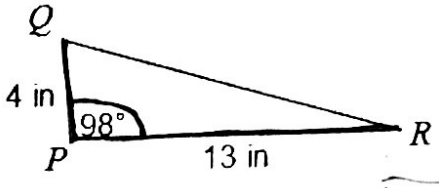
Example 1: Find the area of each triangle. State if there is not enough information given.

a)



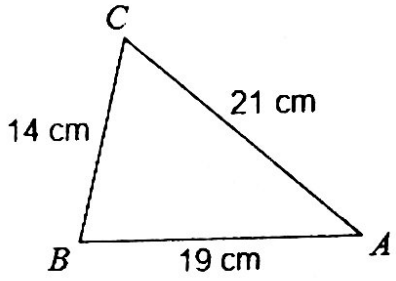
$A = \frac{1}{2}(12)(8)\sin 76 = 46.57 \text{ km}^2$

b)



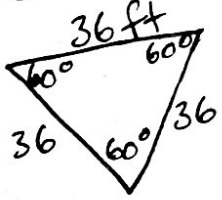
$A = \frac{1}{2}(4)(13)\sin 98 = 25.75 \text{ in}^2$

c)



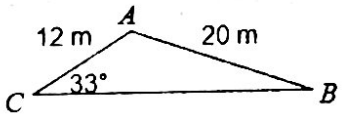
No angle, not enough information

d) Jaron is building a triangular garden in his yard. Each side of the triangle is the same. If the amount of fence he uses on the garden is 36 feet, find the area of the garden he is building.

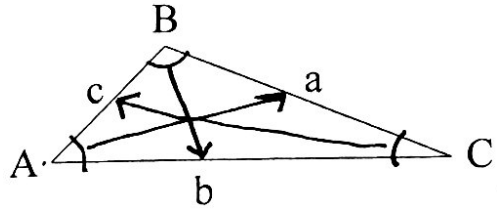


$A = \frac{1}{2}(36)(36)\sin 60 = 561.18 \text{ ft}^2$

So let's go back to the triangle we saw at the beginning of the notes. We cannot use right triangle trigonometry to find the missing sides of this triangle because it's ... Not a right triangle!



LAW OF SINES:



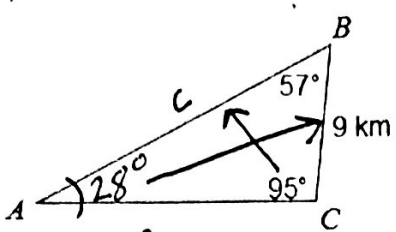
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

- 2 angles & one side
- 2 sides & non-included angle

***The law of sines **ONLY** works when:

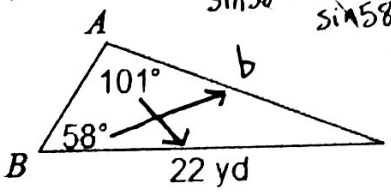
Example 2: Find each measure indicated. If there is not enough information, write NEI.

a) Find AB



$\sin 95 \cdot \frac{c}{\sin 95} = \frac{9}{\sin 28} \cdot \sin 95$ $\quad c = \frac{9\sin 95}{\sin 28} = \boxed{19.10 \text{ km}}$

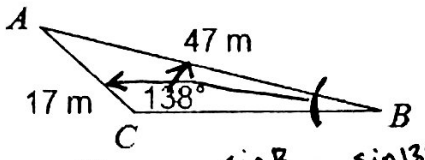
b) Find AC



$\sin 58 \cdot \frac{b}{\sin 58} = \frac{22}{\sin 101} \cdot \sin 58$ $\quad b = \frac{22\sin 58}{\sin 101}$

$C = \boxed{19.01 \text{ yd}}$

c) Find $m\angle B$



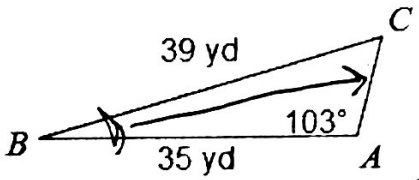
$\frac{17}{\sin B} = \frac{47}{\sin 138}$ $\quad \frac{\sin B}{17} = \frac{\sin 138}{47} \cdot 17$

$\sin B = \frac{17\sin 138}{47}$

$B = \sin^{-1}\left(\frac{17\sin 138}{47}\right)$

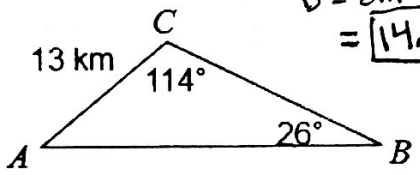
$= \boxed{14.01°}$

d) Find $m\angle B$.

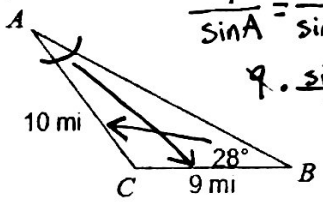


Not enough information

e) Find AB



f) Find $m\angle A$



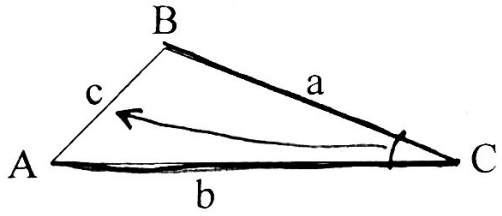
$\frac{9}{\sin A} = \frac{10}{\sin 28}$

$9 \cdot \frac{\sin A}{9} = \frac{\sin 28}{10} \cdot 9$

$\sin A = \frac{9\sin 28}{10}$

$A = \sin^{-1}\left(\frac{9\sin 28}{10}\right) = \boxed{24.99°}$

LAW OF COSINES:

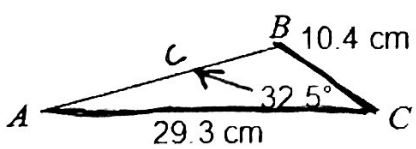


$c^2 = a^2 + b^2 - 2ab\cos C$

- 3 sides
- 2 sides & the included angle

Example 1: Find the missing piece of each triangle:

a) Find AB



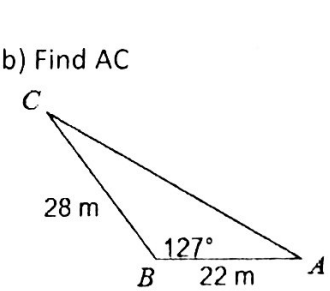
$c^2 = (29.3)^2 + (10.4)^2 - 2(29.3)(10.4)\cos 32.5$

$c^2 = 858.49 + 108.16 - 609.44\cos 32.5$

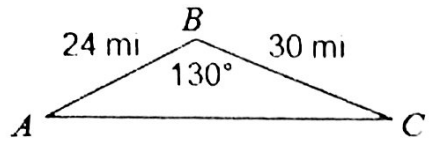
$\sqrt{c^2} = \sqrt{452.65}$

$c = \boxed{21.28 \text{ cm}}$

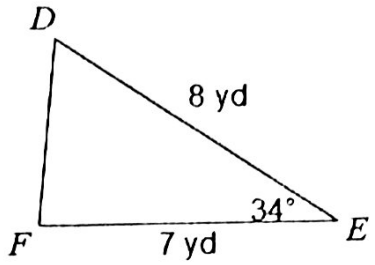
b) Find AC



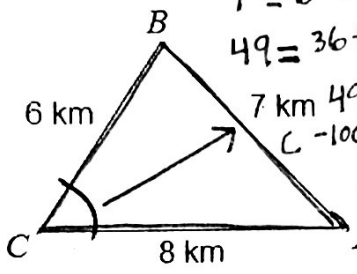
c) Find AC



d) Find DF



e) Find $m\angle C$



$7^2 = 6^2 + 8^2 - 2(6)(8)\cos C$

$49 = 36 + 64 - 96\cos C$

$49 = 100 - 96\cos C$

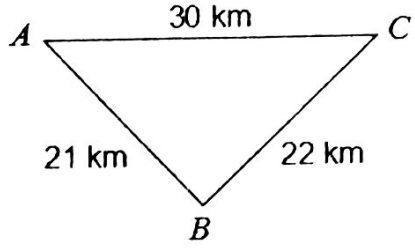
$-100 \quad -100$

$\frac{-51}{-96} = \frac{-96\cos C}{-96}$

$\cos C = \frac{51}{96}$

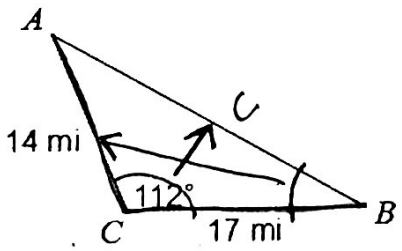
$C = \cos^{-1}\left(\frac{51}{96}\right) = \boxed{57.91^\circ}$

f) Find $m\angle A$

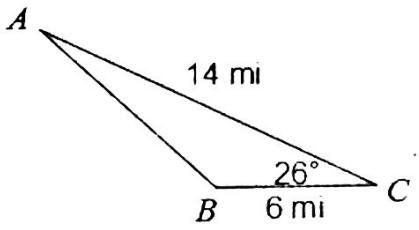


Example 2: Use the Law of Cosines and the Law of Sines. Find each measurement indicated. Round your answers to the nearest tenth.

a) Find $m\angle B$



b) Find $m\angle A$



Law of Sines	Law of Cosines
Use When:	Use When: