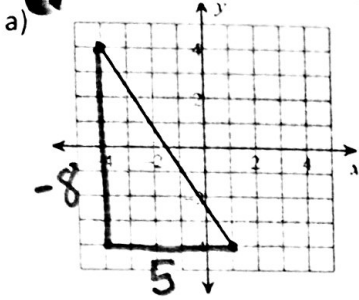


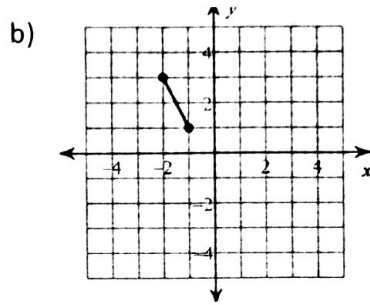
9.4: Distance, Midpoint, and Parallelograms

Distance and Midpoint

1) Find the distance between the following two points:



$$\begin{aligned} (-8)^2 + 5^2 &= c^2 \\ 64 + 25 &= c^2 \\ 89 &= c^2 \\ \boxed{\sqrt{89}} &= c \end{aligned}$$



c) (4, -3) and (-8, -6)

$$-12 \left\langle \begin{array}{c|c} x & y \\ \hline 4 & -3 \\ -8 & -6 \end{array} \right\rangle -3$$

$$\begin{aligned} (-12)^2 + (-3)^2 &= c^2 \\ 144 + 9 &= c^2 \\ \sqrt{153} &= \sqrt{c^2} \\ \boxed{3\sqrt{17}} &= c \end{aligned}$$

d) (-7, 6) and (-5, 2)

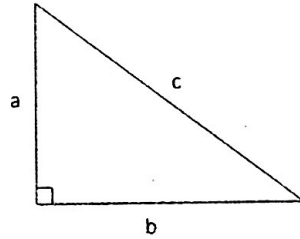
$$2 \left\langle \begin{array}{c|c} x & y \\ \hline -7 & 6 \\ -5 & 2 \end{array} \right\rangle -4$$

$$\begin{aligned} 2^2 + (-4)^2 &= c^2 \\ 4 + 16 &= c^2 \\ \sqrt{20} &= \sqrt{c^2} \\ \boxed{2\sqrt{5}} &= c \end{aligned}$$

To find the distance between two points, we can use:

The Pythagorean Theorem

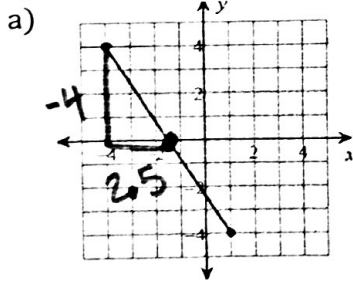
$$a^2 + b^2 = c^2$$



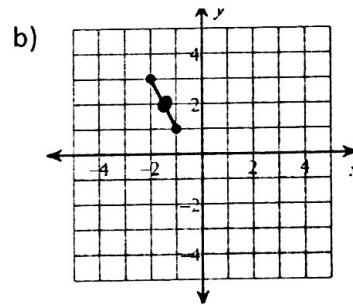
What is the midpoint of a segment?

The middle point of a segment

2) Find the midpoint between the following two points:



$$(-1.5, 0)$$



$$(-1.5, 2)$$

c) (4, -1) and (10, -5)

$$6 \left\langle \begin{array}{c|c} x & y \\ \hline 4 & -1 \\ 10 & -5 \end{array} \right\rangle -4$$

$$\begin{aligned} \frac{6}{2} &= 3 & \frac{-4}{2} &= -2 \\ (4, -1) & & +3, -2 & \\ \boxed{(7, -3)} & & & \end{aligned}$$

d) (-7, 6) and (-5, 2)

$$2 \left\langle \begin{array}{c|c} x & y \\ \hline -7 & 6 \\ -5 & 2 \end{array} \right\rangle -4$$

$$\begin{aligned} \frac{2}{2} &= 1 & \frac{-4}{2} &= -2 \\ (-7, 6) & & +1, -2 & \\ \boxed{(-6, 4)} & & & \end{aligned}$$

find the midpoint:

Find the distances between x's & y's, cut in half, move that far from original point

3) Find the other endpoint of the segment.

a) Endpoint: (4, -3) Midpoint: (-8, -6)

$$\begin{array}{c} x \mid y \\ -12 < \begin{array}{c} 4 \\ -8 \end{array} \mid \begin{array}{c} -3 \\ -6 \end{array} > -3 \\ -12 < \begin{array}{c} -20 \\ -9 \end{array} \mid \begin{array}{c} -3 \\ -6 \end{array} > -3 \end{array}$$

Find the distance, then continue that distance to the other endpoint

$$\boxed{(-20, -9)}$$

b) Endpoint: (-7, 6) Midpoint: (-5, 2)

$$\begin{array}{c} x \mid y \\ 2 < \begin{array}{c} -7 \\ -5 \end{array} \mid \begin{array}{c} 6 \\ 2 \end{array} > -4 \\ 2 < \begin{array}{c} -3 \\ -2 \end{array} \mid \begin{array}{c} 6 \\ 2 \end{array} > -4 \end{array}$$

$$\boxed{(-3, -2)}$$

How would you find a distance that was $\frac{1}{4}$ the distance from an endpoint?

Divide the total distance by 4

4) Find the coordinates of the following points:

a) The endpoints of a segment are (4, 5) and (4, -11). Find the coordinates of the point on the segment that is $\frac{1}{4}$ the distance from (4, 5).

$$\begin{array}{c} x \mid y \\ 0 < \begin{array}{c} 4 \\ 4 \end{array} \mid \begin{array}{c} 5 \\ -11 \end{array} > -16 \end{array}$$

$$\begin{array}{c} (4, 5) \\ +0 \quad -4 \\ \hline \boxed{(4, 1)} \end{array}$$

$$\frac{0}{4} = 0 \quad \frac{-16}{4} = -4$$

b) Find the coordinates of the point on the segment that is $\frac{1}{3}$ of the distance from (4, 5).

$$\begin{array}{c} x \mid y \\ 0 < \begin{array}{c} 4 \\ 4 \end{array} \mid \begin{array}{c} 5 \\ -11 \end{array} > -16 \end{array}$$

$$\begin{array}{c} (4, 5) \\ +0 \quad -5.3 \\ \hline \boxed{(4, -0.3)} \end{array}$$

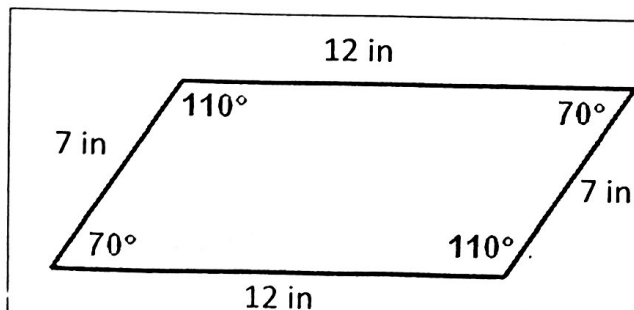
$$\frac{0}{3} = 0 \quad \frac{-16}{3} = -5.3$$

Parallelograms

What does it mean for two lines to be parallel? What does that mean about the slope of the lines?

The lines never touch, their slopes are the same

Below is a diagram of a parallelogram. After studying the diagram, see if you can fill out the four properties of parallelograms.

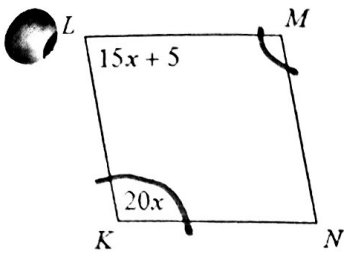


Properties of Parallelograms

- 1) Opposite sides are parallel.
- 2) Opposite sides are congruent.
- 3) Opposite angles are congruent.
- 4) Angles next to each other are supplementary.

3) Find the value of x and the measurement indicated in each parallelogram.

a) Find $m\angle M$



$$15x + 5 + 20x = 180$$

$$35x + 5 = 180$$

$$\quad -5 \quad -5$$

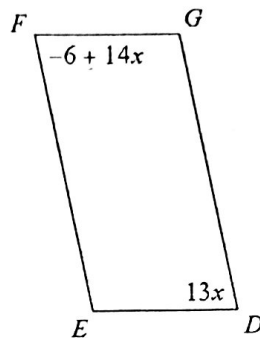
$$\frac{35x}{35} = \frac{75}{35}$$

$$x = 5$$

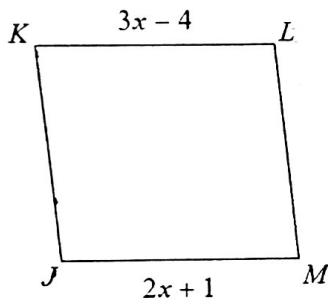
$$m\angle M \cong m\angle K = 20(5)$$

$$= \boxed{100^\circ}$$

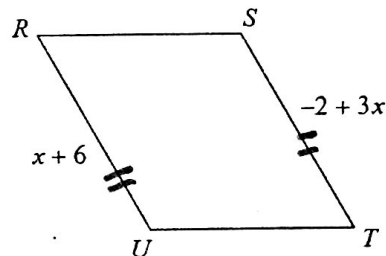
b) Find $m\angle E$



c) Find KL



d) Find ST



$$x + 6 = -2 + 3x$$

$$-x + 2 \quad +2 \quad -x$$

$$\frac{8}{2} = \frac{2x}{2}$$

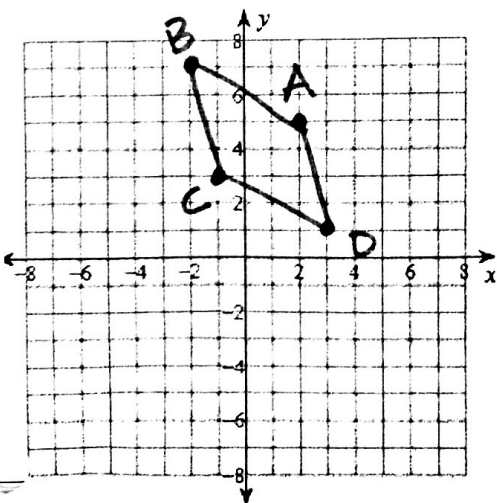
$$4 = x$$

$$\overline{ST} = -2 + 3(4)$$

$$= -2 + 12$$

$$= \boxed{10 \text{ units}}$$

4) Quadrilateral ABCD has the following points: A(2, 5), B(-2, 7), C(-1, 3), D(3, 1). Plot these points on the grid below.



Do you think the quadrilateral is a parallelogram? Why or why not?

Yes, opposite sides are parallel

Show that your thinking is right or wrong:

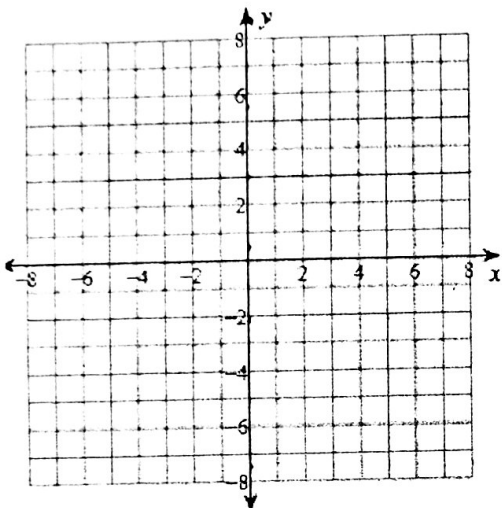
$$\overline{BC}: m = \frac{-4}{1} = -4 \quad \overline{BA}: m = \frac{-2}{4} = -\frac{1}{2}$$

$$\overline{AD}: m = \frac{-4}{1} = -4 \quad \overline{CD}: m = \frac{-2}{4} = -\frac{1}{2}$$

Since opposite sides have the same slope, they are parallel, which means the shape is a parallelogram.

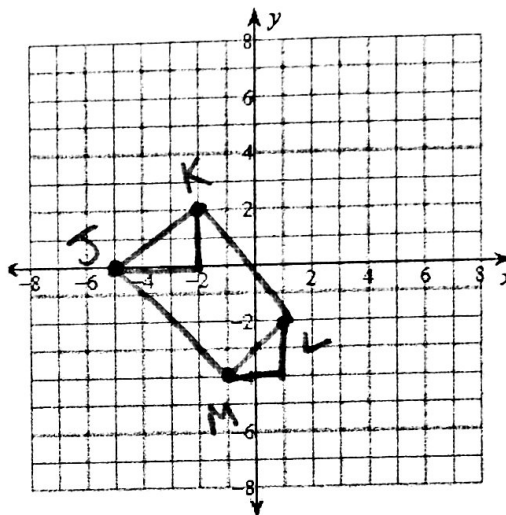
Based on the properties of parallelograms that we explored earlier, we know that parallelograms have opposite sides that are parallel and opposite sides that are congruent. If we can prove either one of these things about a quadrilateral, then we will have proven that it is a parallelogram.

5) Quadrilateral QRST has the following points: Q(3,0), R(1,1), S(4,5), T(2,3). Prove or disprove by slope that the quadrilateral is or is not a parallelogram.



Proof:

6) Quadrilateral JKLM has the following points: J(-5,0), K(-2,2), L(1,-2), M(-1,-4). Prove or disprove by distance that the quadrilateral is or is not a parallelogram.



Proof:

$$\overline{JK}: 3^2 + 2^2 = c^2$$

$$9 + 4 = c^2$$

$$13 = c^2$$

$$\sqrt{13} = c$$

$$\overline{ML}: 2^2 + 2^2 = c^2$$

$$4 + 4 = c^2$$

$$8 = c^2$$

$$\sqrt{8} = c = 2\sqrt{2}$$

Since opposite sides are not congruent, the shape is not a parallelogram.