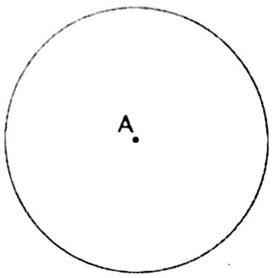
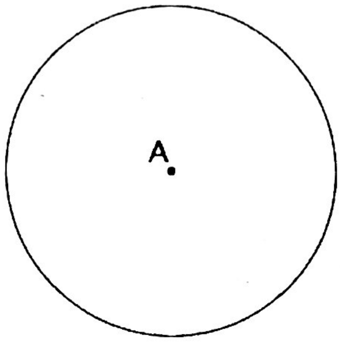
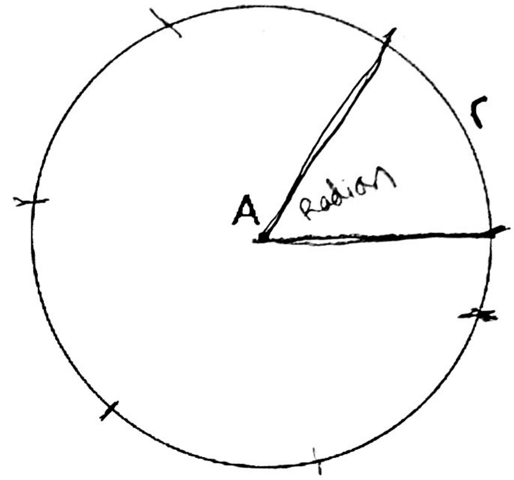
9.3 Radians

What is a radian? To find out, cut a pipe cleaner so you have a piece that is the size of the radius. Take that piece and wrap it around the edge of your circle, marking where it ends. Work with people at your table so there is at least one person working on each size of circle.







* All of the info holds true no matter what size circle you have

1) How many radius lengths did it take to get around the circle? 6 & a little bit ($6.28/2\pi$)

2) Recall that the circumference of a circle is $2\pi r$, where r is the radius of the circle. Where do you think this formula came from? (Hint: What is the value of 2π?)

   $2\pi = 6.28$ It takes 2π radius lengths to get around the circle

3) Draw a segment connecting the ends of a radius length around the edge of a circle to the center of the circle. The angle that you have just created is a **radian.** Describe what a radian is in your own words.

   Radian: Angle that is created when the arc length is the radius length

4) How many radians does it take to make a full rotation? How many degrees? $2\pi / 360°$

5) Write a ratio that compares radians to degrees. Be sure to simplify the ratio. $\pi : 180$

6) How many radians is 60°? How about 270°?

   $\frac{\pi}{3} : \frac{180}{3}$ $\quad \boxed{\frac{\pi}{3}} : 60°$

7) How many degrees is $\frac{\pi}{2}$ radians? How about $\frac{11\pi}{6}$ radians?

   $\frac{\pi}{2} : \frac{180}{2}$ $\quad \boxed{90°}$

   $\frac{11\pi}{6} : \frac{180 \cdot 11}{6}$ $\quad \boxed{330°}$

Convert radians to degrees	Convert degrees to radians
Multiply by $\frac{180}{\pi}$	Multiply by $\frac{\pi}{180}$

In calculator, do not include π when you multiply & reduce the fraction. Just add π to your fully reduced fraction.

8) Convert each degree measure to radians. Leave it as a reduced fraction.

a. $225° \cdot \frac{\pi}{180}$ $\quad \frac{225\pi}{180} = \boxed{\frac{5\pi}{4}}$

b. $315° \cdot \frac{\pi}{180}$ $\quad \boxed{\frac{7\pi}{4}}$

c. $-210° + 360 = 150°$ $\quad 150 \cdot \frac{\pi}{180}$ $\quad \frac{150\pi}{180} = \boxed{\frac{5\pi}{6}}$

9) Convert each radian measure to degrees.

a. $\frac{3\cancel{\pi}}{4} \cdot \frac{180}{\cancel{\pi}}$ $\quad \boxed{135°}$

b. $\frac{7\cancel{\pi}}{6} \cdot \frac{180}{\cancel{\pi}}$ $\quad \boxed{210°}$

c. $\frac{5\cancel{\pi}}{3} \cdot \frac{180}{\cancel{\pi}}$ $\quad \boxed{300°}$

Now go back to the unit circle in the 9.2 notes. Fill in the radian measure for each angle.

10) What are the exact values of each of the following ratios?

$\boxed{\begin{array}{l} \sin\theta = y \\ \cos\theta = x \\ \tan\theta = \frac{y}{x} \end{array}}$

a) $\sin\frac{\pi}{6}$ (y) $= \boxed{\frac{1}{2}}$

b) $\cos\frac{5\pi}{6}$ (x) $\boxed{-\frac{\sqrt{3}}{2}}$

c) $\tan\frac{3\pi}{4}$ $\left(\frac{y}{x}\right)$ $\frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = \boxed{-1}$

d) $\tan\frac{4\pi}{3}$ $\left(\frac{y}{x}\right)$ $\frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\frac{\sqrt{3}}{\cancel{2}} \cdot -\frac{\cancel{2}}{1} = \boxed{\sqrt{3}}$

e) $\sin\frac{\pi}{2}$ (y) $\boxed{1}$

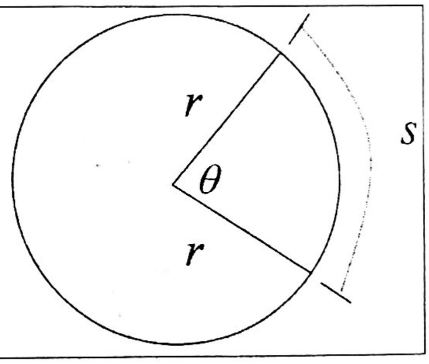
f) $\cos -\frac{\pi}{6}$ (x) $\cos\frac{11\pi}{6}$ $\boxed{\frac{\sqrt{3}}{2}}$

LENGTH OF AN INTERCEPTED ARC:

For a circle with radius r and a central angle θ (in radians), the length of s of the intercepted arc is:

$$s = r\theta$$

arc length → s; length of radians → r; θ ← how many radius lengths it takes to make that angle/arc



11) a. Find the length of s. Exact Approximate

$s = 3 \cdot \frac{5\pi}{6} = \boxed{\frac{15\pi}{6} \approx 7.85 \text{ inches}}$

b. Find the length of b.

$b = \cancel{3} \cdot \frac{2\pi}{\cancel{3}} = \boxed{2\pi \approx 6.28 \text{ inches}}$

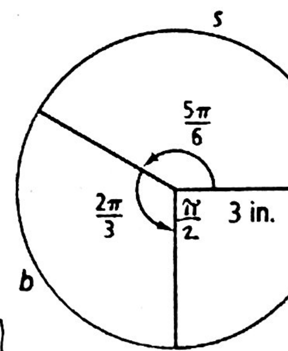
c. Find the length of the missing piece of the circumference.

Missing Angle

$\frac{5\pi}{6} + \frac{2\pi}{3} = \frac{5\pi}{6} + \frac{4\pi}{6} = \frac{9\pi}{6} = \frac{3\pi}{2}$

$2\pi - \frac{3\pi}{2} = \frac{4\pi}{2} - \frac{3\pi}{2} = \frac{\pi}{2}$

$3 \cdot \frac{\pi}{2} = \boxed{\frac{3\pi}{2} \approx 4.71 \text{ inches}}$



12) How would the arc length change if the radius was doubled?

$s = r\theta$ The arc length would double.

$\underline{2}s = \underline{2}r\theta$

If you multiply one side by 2, you have to multiply the other side by 2.