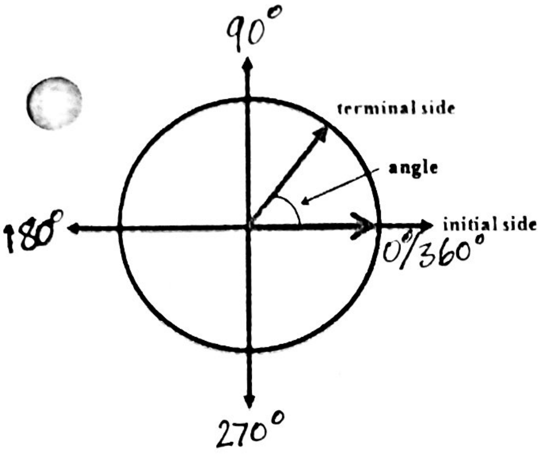
9.2 Angles and the Unit Circle



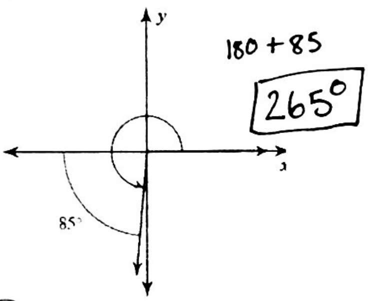
Today we will start looking at angle measures around a coordinate plane. An angle is in **standard position** when the vertex is at the point _(0,0)_. The ray on the _X_-axis is called the **initial side** and the ray on the _y_-axis is called the **terminal side.**

Positive angle measures will be in a counter-clockwise rotation.
Negative angle measures will be in a clockwise rotation.

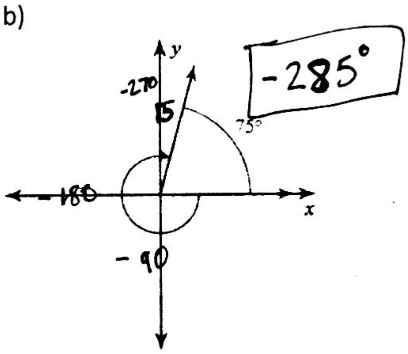
How many degrees would a full rotation be? _360°_
How many degree would half a full rotation be? _180°_

1) Determine the measure of each angle.

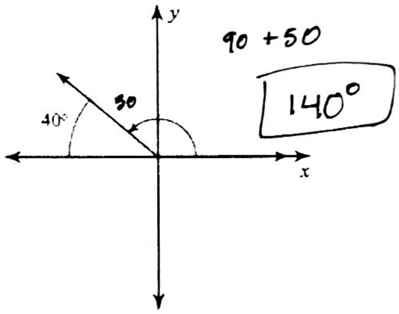
a)



b)

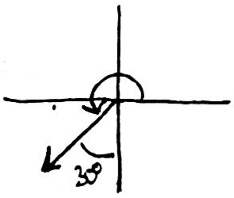


c)

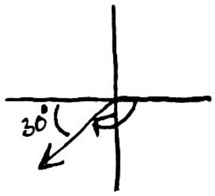


2) Draw each angle. What quadrant is the terminal side in?

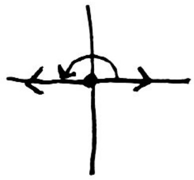
a) 240°



b) −150°

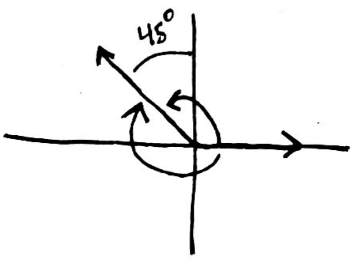


c) 180°



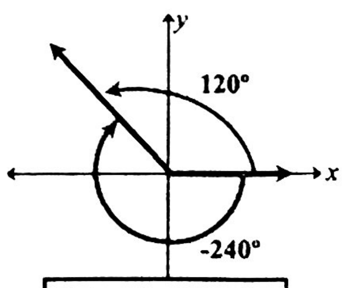
* Be sure your drawing include the direction & the amount between the closest axis

3) Draw 135° and −225° on the same coordinate plane. What do you notice?



They are the same!
(Coterminal angles)

Coterminal angles are angles that are in the same position on the coordinate plane. Coterminal angles can be any angle measure, including those that are greater than the standard 360° rotation.



Coterminal Angles

Strategy for finding coterminal angles	+/- 360°

4) List three coterminal angles for each of the following. Make sure at least one of them is negative.

a) $60° + 360$ $420°$ $780°$

$60 - 360$ $-300°$

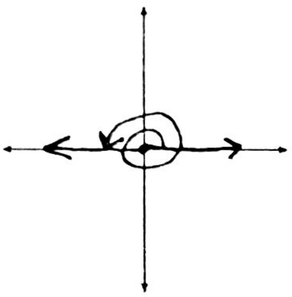
b) $500° - 360$ $140°$ $-220°$ $-580°$

c) $-25° + 360$ $335°$ $695°$

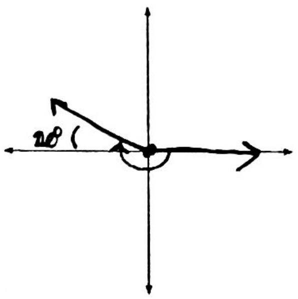
$-25° - 360$ $-385°$

5) Draw each angle on the coordinate plane.

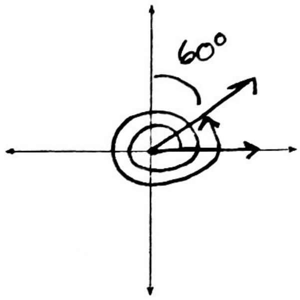
a) $540° - 360 = 180$



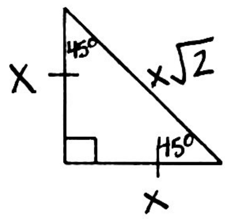
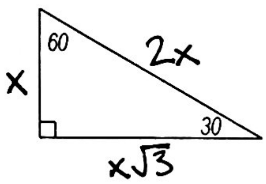
b) $-200° + 360 = 160$



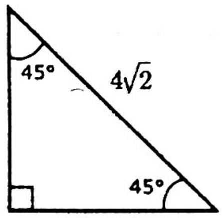
c) $750° - 360 - 360 = 30$

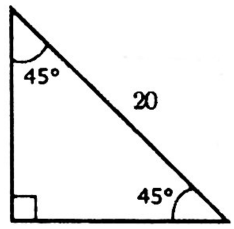


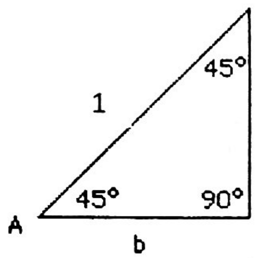
* Draw full rotation path

45-45-90 Triangle	30-60-90 Triangle
	

6) Find the missing lengths of the triangles.

a. 

b. 

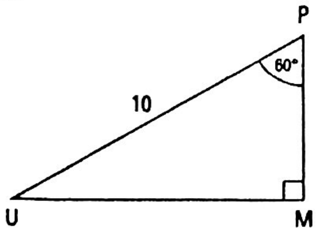
c. 

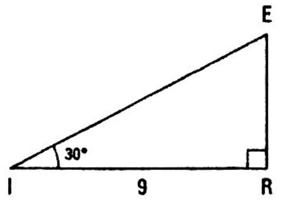
$\frac{1}{\sqrt{2}} = \frac{x\sqrt{2}}{\sqrt{2}}$

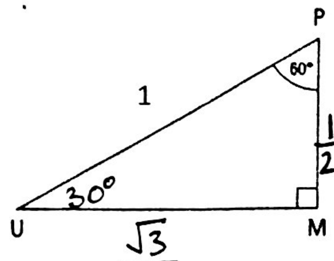
$x = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$x = \frac{\sqrt{2}}{2}$

$a = \frac{\sqrt{2}}{2}, \quad b = \frac{\sqrt{2}}{2}$

d. 

e. 

f. 

$\frac{1}{2} = \frac{2x}{2}$

$\frac{1}{2} = x$

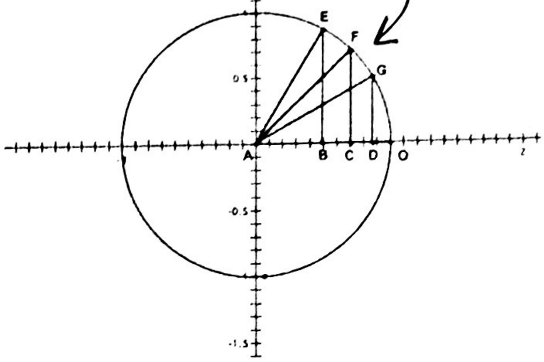
$\frac{\sqrt{3}}{2}$

$\frac{1}{2} \cdot \sqrt{3} = \frac{\sqrt{3}}{2}$

All (x, y) coordinates in the unit circle come from the 45-45-90 & 30-60-90 triangles with a radius of 1.

The Unit Circle

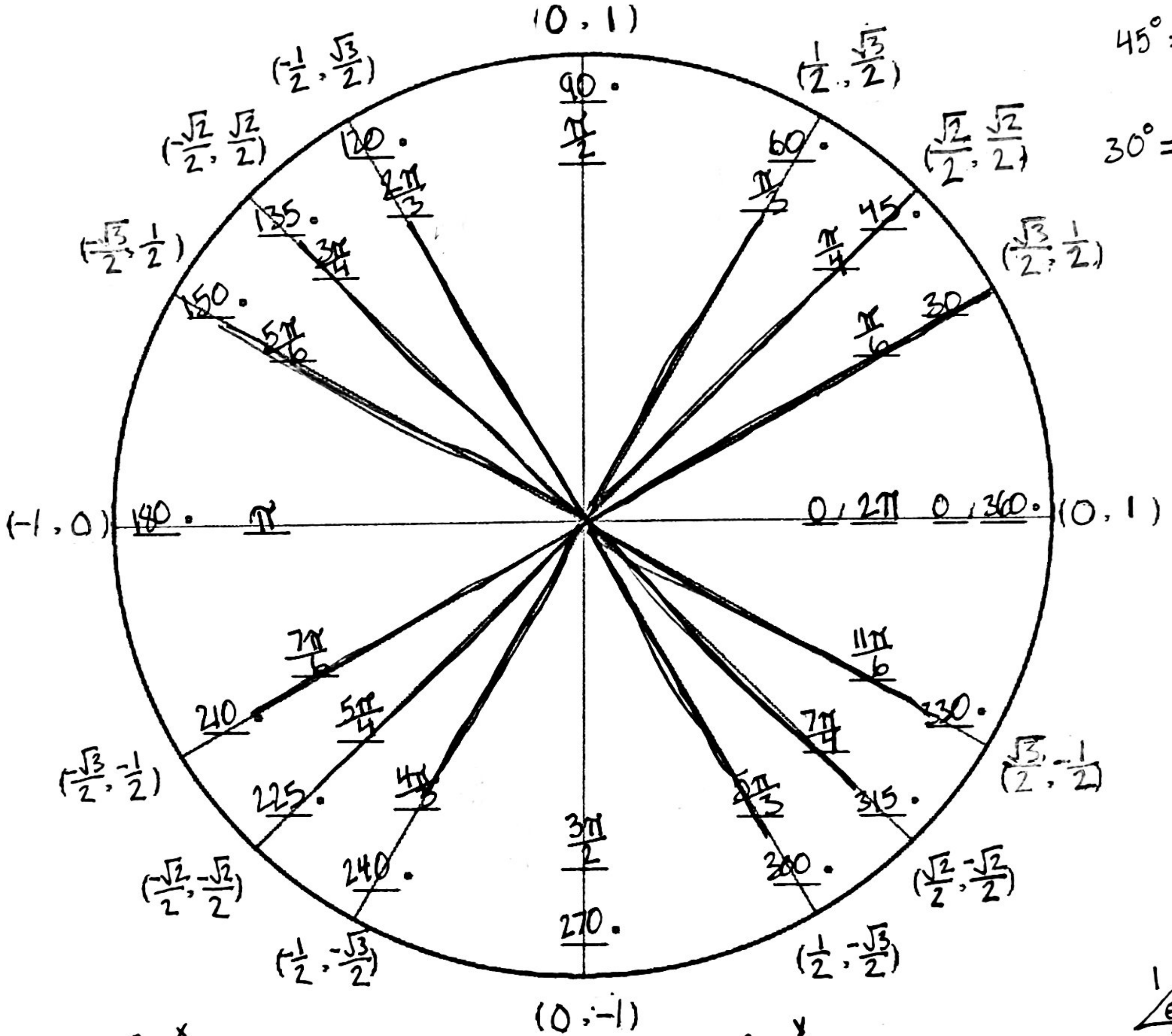
The unit circle is a circle with $\boxed{\text{radius of 1.}}$ It is segmented into 3 different triangles per quadrant, with interior angles of 30°, 45°, and 60°. The figure to the right illustrates this concept for the first quadrant.



Given the special relationships that we have been exploring, see if you can figure out the missing information for the unit circle below.

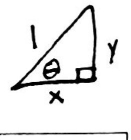
$45° = \frac{\pi}{4}$

$30° = \frac{\pi}{6}$



$\cos\theta = \frac{x}{1}$

$\sin\theta = \frac{y}{1}$



$x = \cos\theta$	$y = \sin\theta$ $\quad \tan\theta = \frac{y}{x}$

7) Find the exact values of each of the following:

a) sin 120°
y-coordinate at 120° $\boxed{\frac{\sqrt{3}}{2}}$

b) cos 270°
x-coordinate at 270° $\boxed{0}$

c) tan 45°
$\frac{\text{y-coordinate at } 45°}{\text{x-coordinate at } 45°}$ $\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \boxed{1}$

d) tan 300°
$\left(\frac{y}{x}\right)$ $\frac{\frac{-\sqrt{3}}{2}}{\frac{1}{2}} = \frac{-\sqrt{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{1} = \boxed{-\sqrt{3}}$

e) sin 135
(y) $\frac{\sqrt{2}}{2}$

f) cos 150°
(x) $\frac{-\sqrt{3}}{2}$