8.5 Solving Exponential and Logarithmic Equations

1) Simplify.

a. $\log_3 81$ 4

b. $\log_5 \frac{1}{125}$ -3

c. $\log_2 64$ 6

d. $\log_8(-16)$ DNE

e. $\log 10$ 1

f. $\ln e^5$ $5\ln e = 5(1) = 5$

g. $\log_2 \frac{1}{8}$ -3
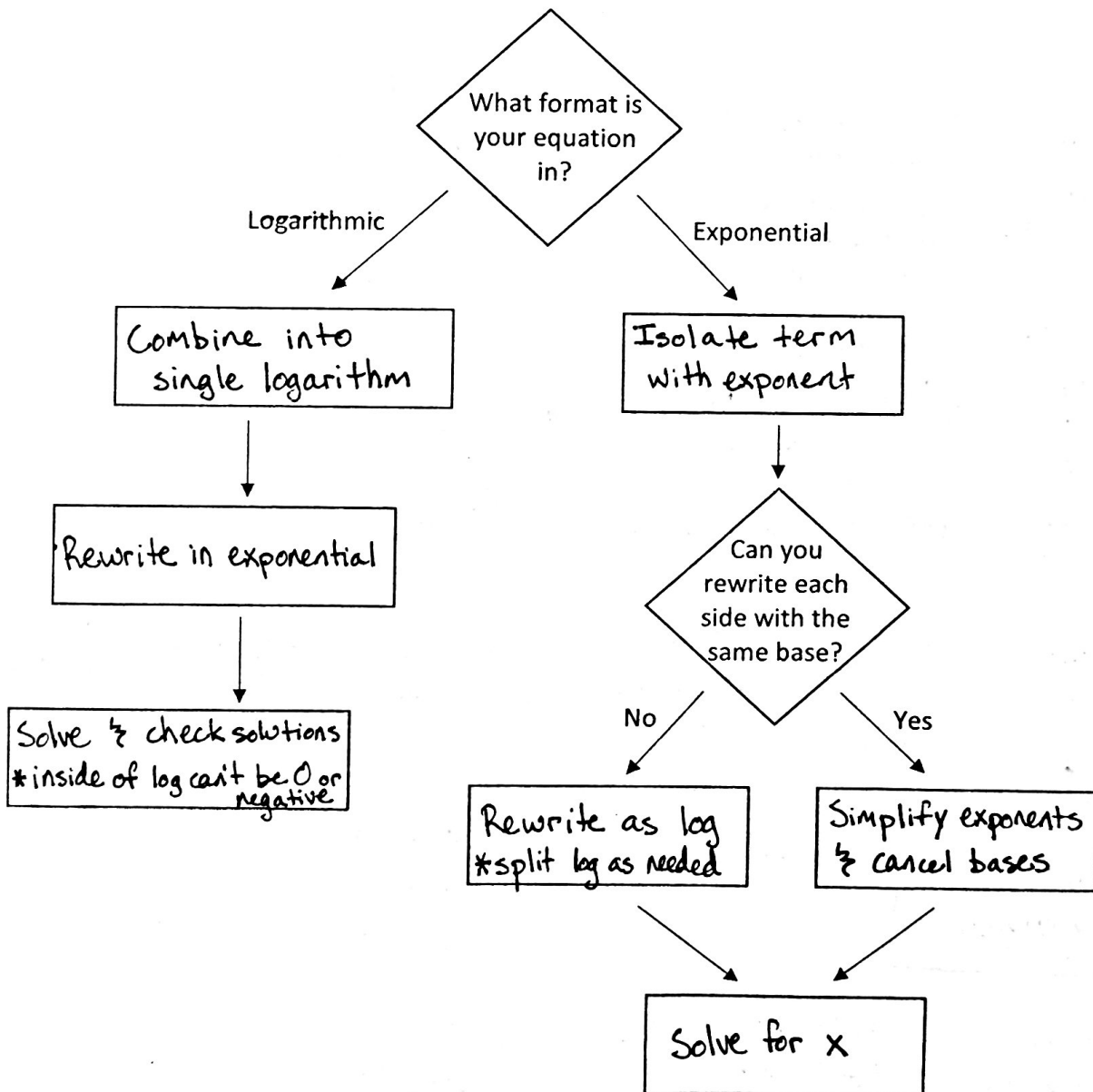
h. $\ln(-2)$ DNE

i. $\log 0$ DNE

* The inside of a log cannot be negative or 0

Solving Exponential and Logarithmic Equations

What format is your equation in?

Logarithmic

Combine into single logarithm

Rewrite in exponential

Solve & check solutions
* inside of log can't be 0 or negative

Exponential

Isolate term with exponent

Can you rewrite each side with the same base?

No

Rewrite as log
* split log as needed

Yes

Simplify exponents & cancel bases

Solve for x

2) Solve for each variable.

a. $27^{3x} = 81$

$(3^3)^{3x} = 3^4$

$3^{9x} = 3^4$

$9x = 4$

$\boxed{x = \frac{4}{9}}$

27 & 81 have a common base of 3, so we'll rewrite them as terms with base of 3

If the bases are the same, the exponents have to be the same

b. $3^{-2x+2} = 81$

$3^{-2x+2} = 3^4$

$-2x+2 = 4$
  $-2 \quad -2$

$\frac{-2x}{-2} = \frac{2}{-2}$

$\boxed{x = -1}$

c. $2^{3x} = 4^{x+1}$

$2^{3x} = (2^2)^{x+1}$

$2^{3x} = 2^{2x+2}$

$3x = 2x+2$
$-2x \quad -2x$

$\boxed{x = 2}$

d. $25^{2x} = 125^{x-1}$

$(5^2)^{2x} = (5^3)^{x-1}$

$5^{4x} = 5^{3x-3}$

$4x = 3x - 3$
$-3x \quad -3x$

$\boxed{x = -3}$

3) Solve each exponential equation. Finals answers should use logs with bases of 10.

a. $5 - 3^x = -40$
   $-5 \qquad -5$

$-3^x = -45$

$3^x = 45$

$\log_3 45 = x$

$\boxed{x = \frac{\log 45}{\log 3}}$

Use logs to solve for exponents

Use change of base formula to write answer using $\log_{10}$

b. $12^{y-2} = 20$

$\log_{12} 20 = y - 2$
$\qquad\quad +2 \quad +2$

$\log_{12} 20 + 2 = y$

$\boxed{y = \frac{\log 20}{\log 12} + 2}$

c. $4e^{2x} + 2 = 16$
   $\qquad -2 \quad -2$

$\frac{4e^{2x}}{4} = \frac{14}{4}$

$e^{2x} = \frac{7}{2}$

$* \ln\frac{7}{2} = 2x$

Split log from here

$\ln 7 - \ln 2 = 2x$

$\boxed{\frac{\ln 7 - \ln 2}{2} = x}$

$* \log_e = \ln$

d. $e^{3x} + 5 = 15$
   $\qquad -5 \quad -5$

$e^{3x} = 10$

$* \frac{\ln 10}{3} = \frac{3x}{3}$

$\boxed{\frac{\ln 10}{3} = x}$

4) Solve each logarithmic equation. Round your answer to the nearest hundredth.

a. $\log(4x - 3) = 2$

$10^2 = 4x - 3$

$100 = 4x - 3$
$+3 \qquad +3$

$\frac{103}{4} = \frac{4x}{4}$

$\boxed{\frac{103}{4} = x}$

Check

$\log\left(4\left(\frac{103}{4}\right) - 3\right)$

$\log(100)$ ✓

b. $\ln(5 - 2x) = 0$

$e^0 = 5 - 2x$

$1 = 5 - 2x$
$-5 \quad -5$

$\frac{-4}{-2} = \frac{-2x}{-2}$

$\boxed{2 = x}$

Check

$\ln(5 - 2(2))$

$\ln(5-4)$

$\ln(1)$ ✓

c. $2\log(x + 1) = 5$

We need to check the solutions to make sure that none of them will make the log negative or 0

Use log properties to combine logs

d. $\ln 2 + \ln 3x = 2$

$\ln(2 \cdot 3x) = 2$

$\ln 6x = 2$

$\frac{e^2}{6} = \frac{6x}{6}$

$\boxed{x = \frac{e^2}{6}}$

e. $\log(x - 3) + \log(x) = 1$

$\log(x(x-3)) = 1$

$\log(x^2 - 3x) = 1$

$10^1 = x^2 - 3x$
$-10 \qquad\qquad -10$

$0 = x^2 - 3x - 10$

$0 = (x-5)(x+2)$

$\boxed{x = 5} \qquad x = -2$

Check

$x = 5 \quad \log(5-3) + \log(5)$ ✓

$x \neq -2 \quad \log(-2-3) + \log(-2)$ ✗

f. $\log 6 - \log 3x = -2$

$\log\frac{6}{3x} = -2$

$10^{-2} = \frac{6}{3x}$

$\frac{1}{100} \times \frac{2}{x}$

$\boxed{x = 200}$