8.4 Properties of Logarithms

Logarithms solve for exponents. This means that properties of logarithms are very similar to properties of exponents.

* Logs must have same base

Recall the following:

Exponential	Logarithmic	Rule
$2^0 = 1 \quad 31^0 = 1$	$\log_2 1 = 0 \quad \log_{31} 1 = 0$	Any log with an inside of 1 is 0
$4^1 = 4 \quad 12^1 = 12$	$\log_4 4 = 1 \quad \log_{12} 12 = 1$	A log where the base and inside match is 1
$b^m \cdot b^n = b^{m+n}$	$\log_b mn = \log_b m + \log_b n$	If there is a product inside a log, split it into two logs added together
$\frac{b^m}{b^n} = b^{m-n}$	$\log_b \frac{m}{n} = \log_b m - \log_b n$	If there is a quotient inside a log, split it into two logs divided

* every subtracted log goes on bottom

1) Expand each logarithm.

a) $\log_4 7x =$

$\log_4 7 + \log_4 x$

b) $\log_3 9y =$

$\log_3 9 + \log_3 y = 2 + \log_3 y$

b) $\log \frac{4x}{y} =$

$\log 4x - \log y$

d) $\log_9 \frac{x^4}{729}$

$\log_9 x^4 - \log_9 729 = \log_9 x^4 - 3$

e) $\ln 4x =$

$\ln 4 + \ln x$

f) $\ln \frac{e^2}{x}$

$\ln e^2 - \ln x = 2 - \ln x$

2) Write each expression as a single logarithm.

a) $\log 4 + \log x + \log y$

$\log 4xy$

b) $\log_4 32 - \log_4 2$

$\log_4 \frac{32}{2} = \log_4 16 = 2$

c) $\log_2 9 - \log_2 3$

$\log_2 \frac{9}{3} = \log_2 3$

d) $\log_2 28 - \log_2 7$

$\log_2 \frac{28}{7} = \log_2 4 = 2$

e) $\ln a - \ln b - \ln c$

$\ln \frac{a}{bc}$

f) $\log a + \log b - \log c$

$\log \frac{ab}{c}$

3) Expand and simplify $\log_3 x^4$.

$\log_3 x \cdot x \cdot x \cdot x = \log_3 x + \log_3 x + \log_3 x + \log_3 x = 4\log_3 x$

Product Property of Logarithms	$\log_b m^n =$
Pop log & drop it	$n \log_b m$

4) Expand each logarithm.

a) $\log_4(x^2y^4)$

$\log_4 x^2 + \log_4 y^4$

$2\log_4 x + 4\log_4 y$

b) $\log_2 \frac{4x^2}{9}$

$\log_2 4x^2 - \log_2 9$

$\log_2 4 + 2\log x - \log_2 9$

c) $\ln 4x^6$

$\ln 4 + \ln x^6$

$\ln 4 + 6\ln x$

d) $\ln\left(\frac{x}{y}\right)^2$

$2\ln\frac{x}{y} = 2\ln x - 2\ln y$

5) Rewrite each expression as a single logarithm.

a) $6\ln x + 5\ln y$

$\ln x^6 + \ln y^5$

$\ln x^6 y^5$

b) $4\log y - (2\log x + 5\log w)$

$\log y^4 - (\log x^2 + \log w^5)$

$\log y^4 - \log x^2 w^5 = \log \frac{y^4}{x^2 w^5}$

Notice that the log button on the calculator won't allow you to change the base of the logarithm. It always assumes the common logarithm. To evaluate a logarithm with any base, use the **CHANGE OF BASE FORMULA**:

Pick whatever base you want (c)

$$\log_b m = \frac{\log_c m}{\log_c b}$$

Usually pick 10 since you can type that in a calculator

Where $b \neq 1, c \neq 1$.

6) What is the value of each expression? Give an exact and an approximate answer.

a) $\log_{81} 27$

$\frac{\log 27}{\log 81} = .75$

b) $\log_5 36$

$\frac{\log 36}{\log 5} \approx 2.23$

c) $\log_8 32$

$\frac{\log 32}{\log 8} \approx 1.67$

d) $\log_4 18$

$\frac{\log 18}{\log 4} \approx 2.08$