8.2 Exponential Models

Financial Models

1) You deposit $100 into a bank account that will accrue 2% interest each year. Fill in the chart below to figure out how much money you will have in your account after 10 years (wait to fill out the last column until we come together as a class).

Year	Starting balance	Interest Calculation	Ending Balance	Simplified Calculation
1	$100	$.02(100) + 100$	$102	$100(1.02)$
2	$102	$.02(102) + 102$	$104.04	$100(1.02)^2$
3	$104.04	$.02(104.04) + 104.04$	$106.12	$100(1.02)^3$
4	$106.12	$.02(106.12) + 106.12$	$108.24	$100(1.02)^4$
5	$108.24	$.02(108.24) + 108.24$	$110.41	$100(1.02)^5$
6	$110.41	$.02(110.41) + 110.41$	$112.62	$100(1.02)^6$
7	$112.62	$.02(112.62) + 112.62$	$114.87	$100(1.02)^7$
8	$114.87	$.02(114.87) + 114.87$	$117.17	$100(1.02)^8$
9	$117.17	$.02(117.17) + 117.17$	$119.51	$100(1.02)^9$
10	$119.51	$.02(119.51) + 119.51$	$121.90	$100(1.02)^{10}$

Simple Interest Equation

$$A = P(1+r)^t$$

A: ending amount
P: principal (starting) balance
r: rate (as a decimal)
t: time

2) Calculate the following balances:

   a. You put $1500 into a college savings account. The account gains 5% annual interest.

   $A = ?$
   $P = 1500$
   $r = 5\% = .05$
   $t = ?$

   $A = 1500(1 + .05)^t$
   $\boxed{A = 1500(1.05)^t}$

   b. Your parents put $300 for you when you were born. The account gains 4% annual interest. How much will be in the account by the time you're 18?

   $A = ?$
   $P = 300$
   $r = 4\% = .04$
   $t = 18$

   $A = 300(1 + .04)^{18}$
   $= 300(1.04)^{18}$
   $= \boxed{607.74}$

   c. $250 is put into an account that gains 6.2% annual interest.

   $A =$
   $P = 600$
   $r = -2.5\% = -.025$
   $t = ?$

   d. $600 is put into an account that ~~gains~~ loses 2.5% annual interest.

   $A = 600(1 - .025)^t$
   $\boxed{A = 600(.975)^t}$

If you ever need to figure out how long it will take to reach a certain amount, it is helpful to use your graphing calculator to do so.

3) You invest $1000 in a savings account that pays 5% annual interest. If you make no additional deposits or withdrawals, how many years will it take for the account to grow to $1500?

Sketch
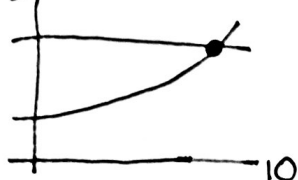
$A = 1500$
$P = 1000$
$r = 5\% = .05$
$t = ?$

$1500 = 1000(1+.05)^t$
$\underline{1500} = \underline{1000(1.05)^t}$
$Y_2 \qquad Y_1$

Plug both sides into graphing calc

[2nd] [Trace] [5]: Intersect

[Enter] × 3 $t = 8.31$ years

4) You invest $500 in a savings account that pays 4.5% annual interest. If you make no additional deposits or withdrawals, how many years will it take for the money that you have in your account to double?

Sketch

Intro to the Natural Number

There are many symbols in mathematics that represent a number, not a variable. One is the natural number, e. $e = \underline{2.71}$. Natural based exponential functions are exponential functions with base e and are useful for describing _continuous_ growth or decay. e: [2nd] [÷]

5) Use a graphing calculator to evaluate the following:

a) e^2 b) e^6 c) e^e

Continuous Growth Equation
$A = Pe^{rt}$ A: ending amount r: rate P: principal (starting) amount t: time

6) Find the amount in a _continuously compounded_ account for the given conditions.

a) principal: $2000, annual interest rate: 5.1%, time: 3 years

$P: 2000$
$r: .051$
$t = 3$

$A = 2000 e^{(.051)(3)}$
$2000 e \wedge (.051 * 3)$
$= \$2330.65$

b) principal: $400, annual interest rate: 7.6%, time: 18 months.

7) Suppose you won a contest at the start of 5th grade that deposited a $3000 in an account that pays 5% annual interest compounded continuously.

a. How much will you have in the account when you enter high school 4 years later?

b. How much would you have in your account when you graduate from high school?

c. How much will you have in your account if your interest rate was 7% instead of 5%?