7.3 Solving Systems by Substitution

We already learned how to solve systems of equations by graphing. Now we are going to solve systems of equations by a method called substitution.

Steps to solving by substitution

1) Get a variable by itself (choose whichever variable is easier to get by itself)

2) Substitute that variable into the other equation
♪♪ Plug it in, plug it in ♪♪

3) Solve

4) Plug result back in to solve for the other variable
(plug in to whatever equation, choose whichever is easier)

1. Practice with Linear Systems of Equations:

a) ① $y = -2x + 4$

$-7x - 3y = -12$

② $-7x - 3(-2x+4) = -12$

③ $-7x + 6x - 12 = -12$
$+12 \quad +12$

$-x = 0$

$x = 0$

④ $y = -2(0) + 4 = 0 + 4 = 4$

$(0, 4)$

b) $-x + y = 1$
$+x \qquad +x$

$-4x + 8y = -24$

① $y = 1 + x$

② $-4x + 8(1+x) = -24$

③ $-4x + 8 + 8x = -24$

$4x + 8 = -24$
$-8 \quad -8$

$\frac{4x}{4} = \frac{-32}{4}$

$x = -8$

④ $x = -8$ $\quad y = 1 + (-8) = -7$

$(-8, -7)$

b) $6x + y = 16$
$-6x \qquad -6x$

$-8x - 3y = -8$

① $y = 16 - 6x$

② $-8x - 3(16 - 6x) = -8$

③ $-8x - 48 + 18x = -8$

$10x - 48 = -8$
$+48 \quad +48$

$\frac{10x}{10} = \frac{40}{10}$

$x = 4$

④ $y = 16 - 6(4)$ $\quad x = 4$
$= 16 - 24$
$= -8$

$(4, -8)$

d) $3x + 7y = 23$

$-4x - y = 11$
$+4x \qquad +4x$

① $\frac{-y}{-1} = \frac{11 + 4x}{-1}$

$y = -11 - 4x$

② $3x + 7(-11 - 4x) = 23$

③ $3x - 77 - 28x = 23$

$-25x - 77 = 23$
$+77 \quad +77$

$\frac{-25x}{-25} = \frac{100}{-25}$

$x = -4$

④ $x = -4$ $\quad y = -11 - 4(-4)$
$= -11 + 16$
$= 5$

$(-4, 5)$

2. Practice with Quadratic Systems of Equations:

a) $y = x^2 - 11x + 28$

① $y = -3x + 12$

② $-3x + 12 = x^2 - 11x + 28$
$+3x \quad -12 \qquad +3x \quad -12$

③ $0 = x^2 - 8x + 16$ $\quad 16x^2$ / $-4 \; -4$

$0 = (x-4)(x-4)$

$x = 4 \quad x = 4$

④ $x = 4$ $\quad y = -3(4) + 12$

$= -12 + 12$

$= 0$

$\boxed{(4, 0)}$

b) $y = 2x^2 + 13x + 15$

$y = x - 1$

c) $x^2 + y^2 = 25$

① $y = 2x - 10$

② $x^2 + (2x-10)^2 = 25$

③ $x^2 + (2x-10)(2x-10) = 25$

$x^2 + 4x^2 - 20x - 20x + 100 = 25$
$\qquad\qquad -25 \quad -25$

$5x^2 - 40x + 75 = 0$

$5(x^2 - 8x + 15) = 0$ $\quad 15x^2$ / $-3 \; -5$

$5(x-3)(x-5) = 0$

$x = 3 \quad x = 5$

$x = 3$ $\quad y = 2(3) - 10$

$= 6 - 10$

$= -4$

$x = 5$ $\quad y = 2(5) - 10$

$= 10 - 10$

$= 0$

$\boxed{(3, -4), (5, 0)}$

d) $x^2 + y^2 = 68$

① $y = -4x$

② $x^2 + (-4x)^2 = 68$

$x^2 + 16x^2 = 68$

$\frac{17x^2}{17} = \frac{68}{17}$

$\sqrt{x^2} = \sqrt{4}$

$x = \pm 2$

If there is more than one x, then there are more than one solution.

$x = 2 \quad y = -4(2) = -8$

$x = -2 \quad y = -4(-2) = 8$

$\boxed{(2, -8), (-2, 8)}$

Methods to solving quadratics		
1) Factoring	2) Quadratic Formula	3) Complete the Square

* Before you start solving, make sure one side is set equal to 0

Tips to solving word problems
1) Identify variables • what are they talking about that you don't know? 2) Write equations • translate English to math 3) Determine what to solve for (one/both of your variables)

3. Word Problems

a) A personal aircraft is traveling with the wind and flies 520 miles in 4 hours. On the return trip, the airplane is traveling against the wind and takes 5 hours to travel the same distance. Find the speed of the airplane in still air.

p = airplane speed
w = wind speed

*speed is mph

$p + w = \frac{520}{4}$ *

$p - w = \frac{520}{5}$

$p + w = 130$

$p - w = 104$

$p = 104 + w$

$(104 + w) + w = 130$

$104 + 2w = 130$

$-104 \quad -104$

$\frac{2w}{2} = \frac{26}{2}$

$w = 13$

$p = 104 + 13$

$p = 117$ mph

b) A field goal is 3 points and the extra point after a touchdown is 1 point. In a recent post-season, Adam Vinatieri of the Indianapolis Colts made a combined total of 21 field goals and extra-point kicks for a total of 49 points. How many filed goals and how many extra-point kicks did he make?

f = # of field goals
e = # of extra-points

$f + e = 21$

$3f + 1e = 49$

$f = 21 - e$

$3(21 - e) + e = 49$

$63 - 3e + e = 49$

$63 - 2e = 49$

$-2e = -14$

$e = 7$

$f = 21 - 7$

$= 14$

14 field goals
7 extra-point kicks

c) The revenue for a production of Wizard of Oz by a theatre group is $y = -50t^2 + 300t$ where t is the ticket price in dollars. The cost for the production is $y = 600 - 50t$. Determine the ticket price that will allow the production to break even. (Hint: a company breaks even when the revenue is equal to the cost)