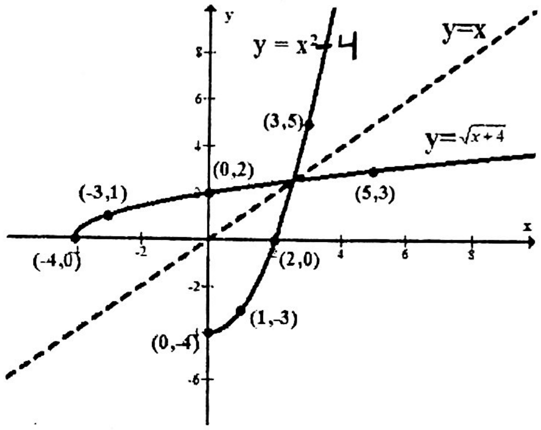
7.3 Inverse Functions

When solving equations, you have been using what we call the *inverse operation* to get the variable by itself. This basically means you are performing the operation that will "undo" another. For example, the inverse operation of addition is subtraction and the inverse operation of multiplication is division.

Much like inverse operations, there are inverse functions which "undo" another function. For example, the inverse function of $f(x) = x^2$ is $\boxed{f^{-1}(x)} = \sqrt{x}$. Today we will explore the relationships between functions and their inverses.

To the right is a diagram of a function and its inverse. What are some patterns that you notice?

x & y coordinates switch
Equations are opposite of each other
Graphs are mirrored over y = x



Theme to inverse relations:
Switch x & y

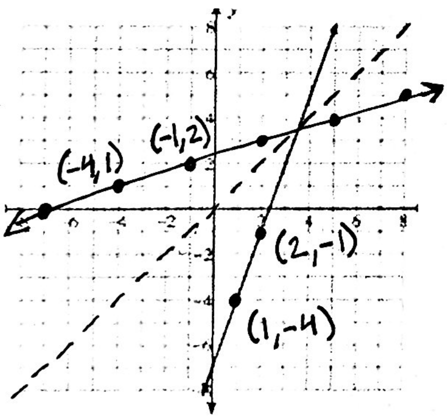
When we graph, this will result in a reflection over the line $y = x$.

1) Graph the inverse of each function.

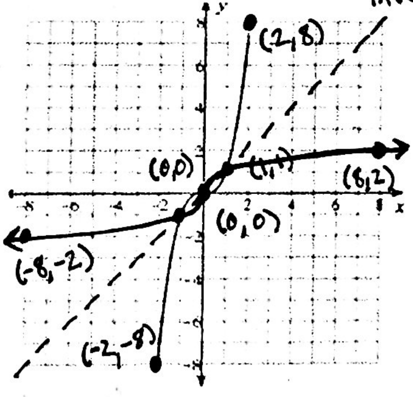
Switch x & y and continue slope

Find points from function, then switch x & y to graph inverse

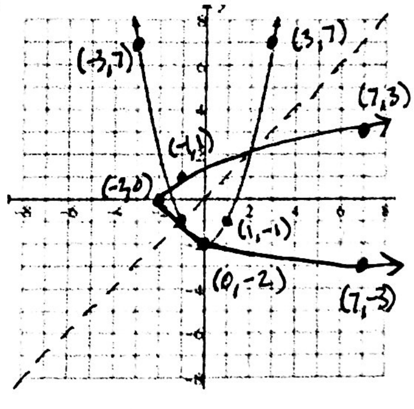
a.



b.



c.



2) State the domain and range of each function and its inverse above.

a. Domain $f(x)$: $(-\infty, \infty)$
Range $f(x)$: $(-\infty, \infty)$
Domain $f^{-1}(x)$: $(-\infty, \infty)$
Range $f^{-1}(x)$: $(-\infty, \infty)$

b. Domain $f(x)$: $(-\infty, \infty)$
Range $f(x)$: $(-\infty, \infty)$
Domain $f^{-1}(x)$: $(-\infty, \infty)$
Range $f^{-1}(x)$: $(-\infty, \infty)$

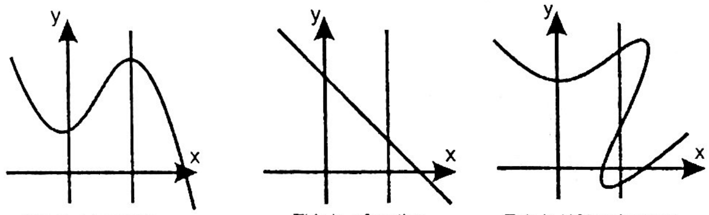
c. Domain $f(x)$: $(-\infty, \infty)$
Range $f(x)$: $[-2, \infty)$
Domain $f^{-1}(x)$: $[-2, \infty)$
Range $f^{-1}(x)$: $(-\infty, \infty)$

Domain: All possible x-values * scan left to right	Range: All possible y-values * scan bottom to top

Because domain and range deal with x and y, the theme of inverses applies here as well.

The domain of $f(x)$ will be the range of $f^{-1}(x)$.
The range of $f(x)$ will be the domain of $f^{-1}(x)$.

Though you may be asked to find the inverse of a function, this does not mean that the inverse itself is a function. To be a function, there must only be one x for every y (meaning one x-value cannot map to two different y-values). A way we check this is with something called **the vertical line test**. Basically if you were to scan a vertical line from left to right on your graph and the vertical line hits more than one point at the same time, then the graph is not a function.



This is a function | This is a function | This is NOT a function

To find an inverse equation, we will again draw from the theme of inverses (switch x & y). The goal is to get y by itself.

3) Find the inverse of each function. Determine if the inverse is a function.

a) $y = x^2 - 1$

Switch x & y, then solve for y

$x = y^2 - 1$
$+1 \quad +1$

$\sqrt{x+1} = \sqrt{y^2}$

$\boxed{\sqrt{x+1} = y}$

b) $y = 4 - 3x$

$x = 4 - 3y$
$+4 \quad +4$

$\dfrac{x+4}{-3} = \dfrac{-3y}{-3}$

$\boxed{\dfrac{-x-4}{3} = y}$

We don't like negatives in denominator, so switch signs of terms on top to make the bottom positive.

c) $y = (1 - 2x)^2 + 5$

$x = (1-2y)^2 + 5$
$-5 \quad -5$

$\sqrt{x-5} = \sqrt{(1-2y)^2}$

$\sqrt{x-5} = 1 - 2y$
$-1 \quad -1$

$\dfrac{\sqrt{x-5} - 1}{-2} = \dfrac{-2y}{-2}$

$\boxed{\dfrac{-\sqrt{x-5} + 1}{2} = y}$

d) $y = \sqrt{x-1} + 5$

$x = \sqrt{y-1} + 5$
$-5 \quad -5$

$(x-5)^2 = (\sqrt{y-1})^2$

$(x-5)^2 = y - 1$
$+1 \quad +1$

$\boxed{(x-5)^2 + 1 = y}$