# 7.2 Piecewise Functions

Up until now a function has been represented by a single equation. In many real-life problems, however, functions are represented by a combination of equations, each corresponding to a part of the domain. Such functions are called **PIECEWISE FUNCTIONS.** For example, the piecewise function given by

$$f(x) = \begin{cases} 2x - 1, & if\ x \le 1 \\ 3x + 1, & if\ x > 1 \end{cases}$$

Is defined by two equations. One equation gives the values of $f(x)$ when $x$ is less than or equal to 1, and the other equation gives the values of $f(x)$ when $x$ is greater than 1.

*To evaluate an equation at a certain x-value of a piecewise function, first determine which equation to plug x into. That means you have to determine which domain your x falls in and plug it into the corresponding equation.

1. a) Evaluate $f(x)$ when (a) $x = 0$ (b) $x = 2$ (c) $x = 4$.

$$f(x) = \begin{cases} x^2 - 1, & if\ x \le 2 \\ 4x + 1, & if\ x > 2 \end{cases}$$

(a) $(0)^2-1$, $0-1=\boxed{-1}$

(b) $(2)^2-1$, $4-1=\boxed{3}$

(c) $4(4)+1$, $16+1=\boxed{17}$

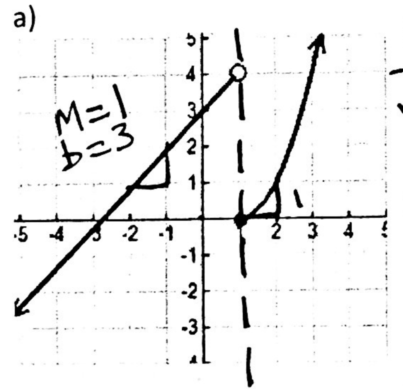
b) Find (a) $f(-4)=\boxed{-3}$ (b) $f(0)=\boxed{-1}$ (c) $f(3)=\boxed{2}$ (d) $f(18)=\boxed{36}$

$$f(x) = \begin{cases} x + 1, & -5 \le x < -1 \\ x - 1, & -1 < x \le 3 \\ 2x, & x > 3 \end{cases}$$

Just plug in x-value into whatever equation has the interval that includes that x-value.

2. What equation would you use to describe the graphs below:

a)



Line

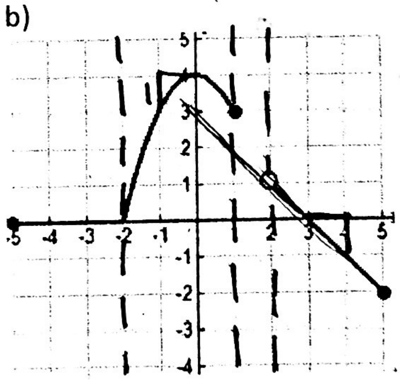
$y = mx + b$

slope, y-int

$M=1$
$b=3$

$$y = \begin{cases} x+3, & x<1 \\ (x-1)^2, & x \ge 1 \end{cases}$$

b)



If you don't know y-int, continue line until you see it

$$y = \begin{cases} 0, & -5 \le x \le -2 \\ -x^2+4, & -2 < x \le 1 \\ -x+3, & 2 < x \le 5 \end{cases}$$

Open hole: $<$, $>$
Closed point: $\le$, $\ge$

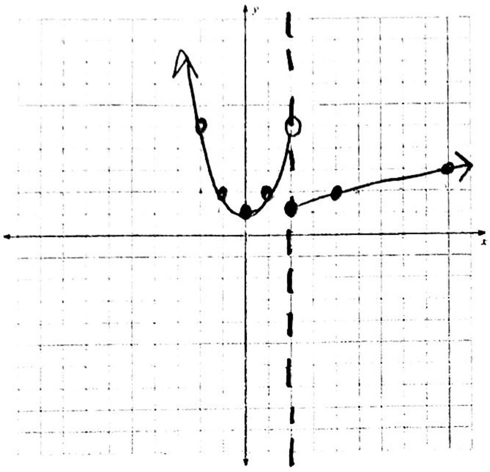
Graphing piecewise functions can be done in one of two ways:

1) Graph all equations and erase any pieces that lie outside of the given domain or
2) Draw lines where the graph changes and only draw the graphs within the boundaries

3. Graph the following piecewise functions.

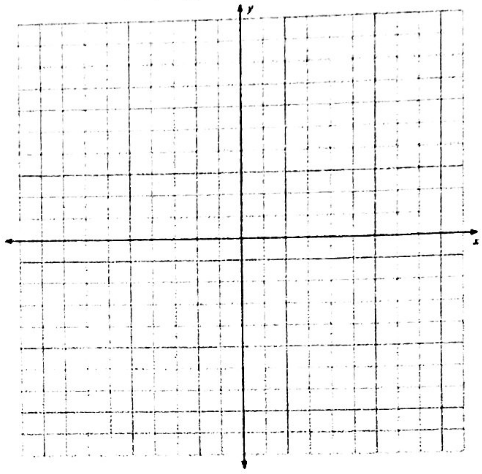
a)

$$y = \begin{cases} x^2 + 1, & x < 2 \\ \sqrt{x}, & x \geq 2 \end{cases}$$



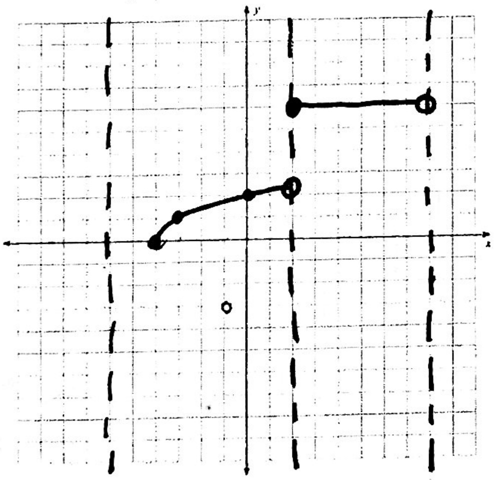
b)

$$y = \begin{cases} \sqrt[3]{x}, & 0 \leq x < 8 \\ -|x|, & 8 \leq x < 10 \end{cases}$$



c)

$$y = \begin{cases} \sqrt{x + 4}, & -6 \leq x < 2 \\ 6, & 2 \leq x < 8 \end{cases}$$

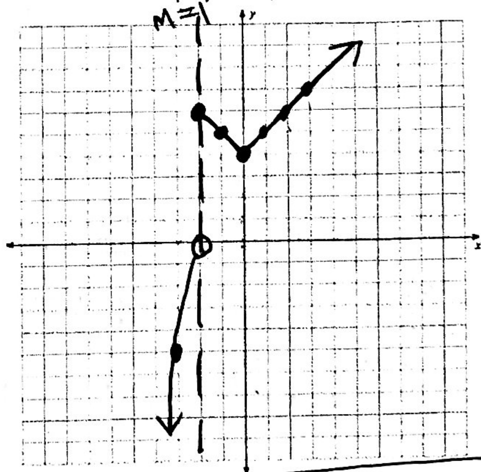


Steps

1) Draw dotted lines wherever the domain changes
2) Graph all equations
3) Erase graphs where they're not supposed to be (including open/closed points)

d)

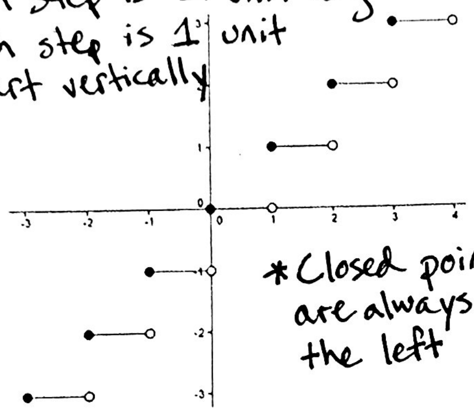
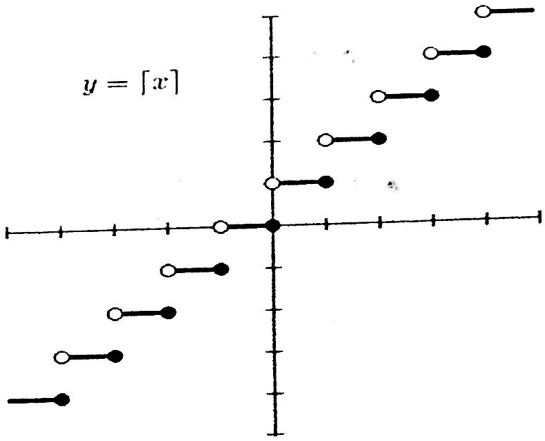
$$f(x) = \begin{cases} -x^2 + 4, & x < -2 \\ |x| + 4, & x \geq -2 \end{cases}$$




Absolute value graphs graph like a normal line that just goes up in both directions, which creates a v-shape.

## Step Functions

A step function is a type of piecewise function that basically looks like a staircase when you graph it.

| Floor Function: $y = \lfloor x \rfloor$ | Ceiling Function: $y = \lceil x \rceil$ |
|---|---|
| Always round down<br>ex: $\lfloor 2.6 \rfloor = 2$<br>$\lfloor -4.1 \rfloor = -5$ | Always round up<br>Ex: $\lceil 3.2 \rceil =$<br>$\lceil -8.9 \rceil =$ |
| - each step is 1 unit long<br>- each step is 1 unit apart vertically<br><br>* Closed points are always on the left | <br> |

4. Write the floor and ceiling functions as piecewise functions on the interval [-2, 2].

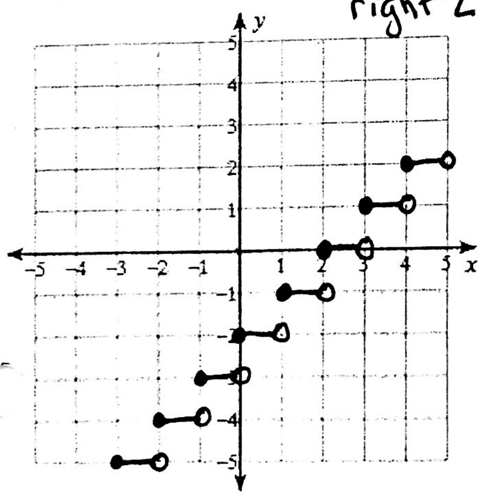
Floor

Ceiling

What is the difference between a floor and ceiling function on a graph?

5. Graph the following functions. Make a behavior table to help you graph.

a. $f(x) = \lfloor x - 2 \rfloor$ Horizontal shift right 2



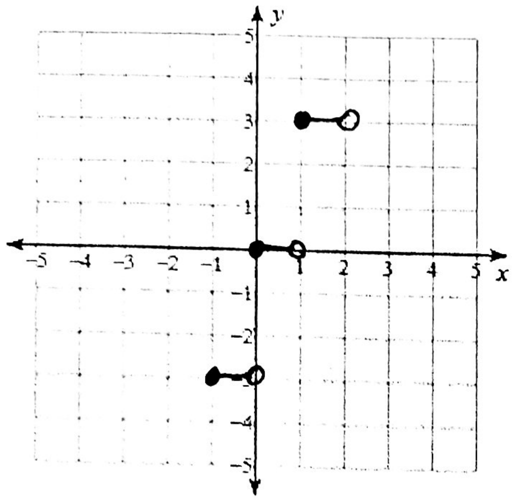
| x | y |
|---|---|
| -1 | -3 |
| -1.5 | -4 |

x = -1 $y = \lfloor -1 - 2 \rfloor$
$y = \lfloor -3 \rfloor$
$= -3$

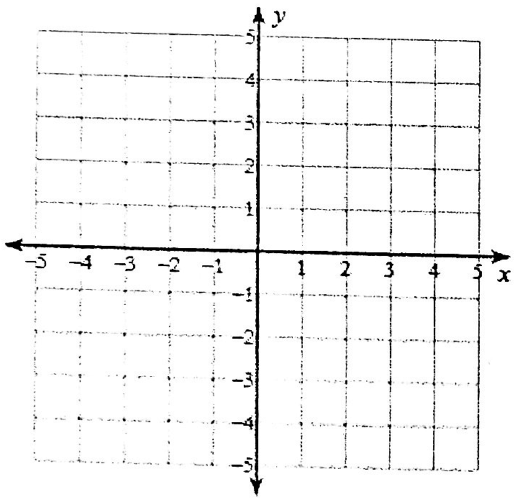
x = -1.5 $y = \lfloor -1.5 - 2 \rfloor$
$y = \lfloor -3.5 \rfloor$
$= -4$

This is just to show you why the function creates step lines. Focus on using graphing transformations based on the parent graph to graph these.

b. $f(x) = 3\lfloor x \rfloor$ Vertical stretch of 3




c. $f(x) = \lceil x \rceil + 2$




d. $f(x) = \lceil 2x \rceil$ Horizontal stretch of 2

- steps are 2 units long