Unit 5.3: Solving Rational Equations

1. Solve each equation.

a. $\frac{x}{4} = \frac{3}{2}$ b. $\frac{2x+1}{2} = \frac{x+2}{5}$ c. $\frac{2}{5} = \frac{x}{21-x}$

* If the bottoms are the same, then the tops have to be the same.

2. Solve each equation.

a) $\frac{1}{x^2} = \frac{3}{x^2} + \frac{4}{x}$ $x \neq 0$

$\frac{1}{x^2} = \frac{3}{x^2} + \frac{4}{x} \cdot \frac{x}{x}$

$\frac{1}{x^2} = \frac{3}{x^2} + \frac{4x}{x^2}$

$* \frac{1}{\cancel{x^2}} = \frac{3+4x}{\cancel{x^2}}$

$1 = 3 + 4x$

$-2 = 4x$

$\boxed{-\frac{1}{2} = x}$ $x \neq 0$

b) * $\frac{x-1}{\cancel{x+2}} = \frac{x^2+2x-3}{\cancel{x+2}}$ $x \neq -2$

$x - 1 = x^2 + 2x - 3$

$-x \quad +1 \qquad -x \quad +1$

$0 = x^2 + x - 2$ $\quad -2x^2$

$0 = (x+2)(x-1)$ $\quad 2 \quad -1$

$x = -2$ $\boxed{x = 1}$

$x = 1$ is the only possible solution since the domain says $x \neq -2$. Therefore $x = -2$ is an extraneous solution.

c) $\frac{x-1}{x^2+3x+2} + \frac{2x}{x+2} = \frac{x-1}{x+1}$ $x \neq -2, -1$

$\frac{x-1}{(x+2)(x+1)} + \frac{2x}{x+2} = \frac{x-1}{x+1}$

$\frac{x-1}{(x+2)(x+1)} + \frac{2x(x+1)}{(x+2)(x+1)} = \frac{(x-1)(x+2)}{(x+1)(x+2)}$

$* \frac{x-1 + 2x(x+1)}{\cancel{(x+2)(x+1)}} = \frac{(x-1)(x+2)}{\cancel{(x+1)(x+2)}}$

$x - 1 + 2x(x+1) = (x-1)(x+2)$

$x - 1 + 2x^2 + 2x = x^2 + x - 2$

$2x^2 + 3x - 1 = x^2 + x - 2$

$-x^2 \quad -x \quad +2 \quad -x^2 \quad -x \quad +2$

$x^2 + 2x + 1 = 0$ $\quad 1x^2$

$1 \quad 1$

$(x+1)(x+1) = 0$

$x = -1 \quad x = -1$ Domain: $x \neq -1$

d) $\frac{1}{2x+8} + \frac{3}{x+4} = \frac{x-4}{x^2+4x}$

$\boxed{\text{No Solution}}$

Steps to Solving Rational Equations

1) Find domain of all fractions (need to factor denominators)
2) Get LCD (least common denominator) by multiplying each fraction by the factor that it's missing
3) Cancel denominators & solve
4) Check solutions with domain