

Unit 5.2: Graphing Rational Functions

You can get a reasonable graph for a rational function by finding all intercepts and asymptotes. Sometimes you will also have to plot a few extra points to get a good sense of the shape of the graph.

1. What is the graph of the rational function: $y = \frac{x^2+x-12}{x^2-4}$

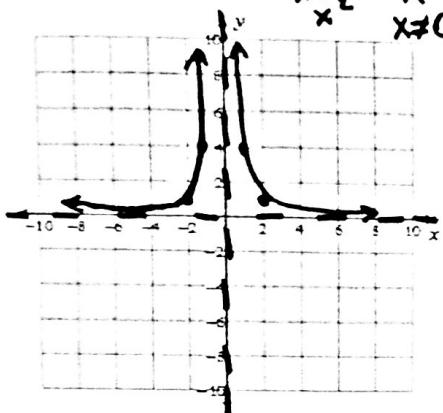
i. Check for Horizontal Asymptotes	Equally weighted: Divide leading coefficients $\frac{1x^2}{1x^2} = 1$ y = 1
ii. Factor both the numerator and denominator. Check for Vertical Asymptotes or Holes in the graph.	$y = \frac{(x+4)(x-3)}{(x+2)(x-2)}$ No holes since nothing cancels $x \neq -2, 2$ Vertical Asymptotes: $x = -2$, $x = 2$
iii. Find the x and y intercepts	x-int: Set numerator equal to 0 $0 = (x+4)(x-3)$ $x = -4, x = 3$ (-4, 0), (3, 0) y-int: Plug in 0 for x $y = \frac{0+0-12}{0-4} = \frac{-12}{-4} = -3$ (0, -3)
iv. Find a few more points. Pick any values of x that will give you a general shape of the graph	$x = -8$ $y = \frac{(-8+4)(-8-3)}{(-8+2)(-8-2)} = \frac{(-4)(-11)}{(-6)(-10)} = \frac{44}{60} = \frac{11}{15}$ $(-8, \frac{11}{15})$ $x = 6$ $y = \frac{(6+4)(6-3)}{(6+2)(6-2)} = \frac{(10)(3)}{(8)(4)} = \frac{30}{32} = \frac{15}{16}$ $(6, \frac{15}{16})$
v. Graph the asymptotes, plot the intercepts and additional points. Use the points to sketch the graph.	<p>Connect points based on boundary of asymptotes</p>

Basically do everything you did in 5.1, but then put it on a graph.

* Graphs cannot cross vertical asymptotes since the values are not part of the domain

* A horizontal asymptote represents the value the function approaches as * gets really big/small (ends of graph). This means you can potentially have a graph that crosses a horizontal asymptote.

a) $y = \frac{4x}{x^3} = \frac{4x}{x^3} = \frac{4}{x^2}$ $x \neq 0$



HA: Bottom heavy, $y=0$

Hole: We would have a hole at $x=0$, but there is a VA there, which is more prominent

VA: $x=0$

x-int: $y=0$ No x-int since no x in numerator

y-int: $y=\frac{4}{0}$ DNE, no y-int

Other: $x=2 \quad y=\frac{4}{(2)^2}=\frac{4}{4}=1 \quad (2,1)$

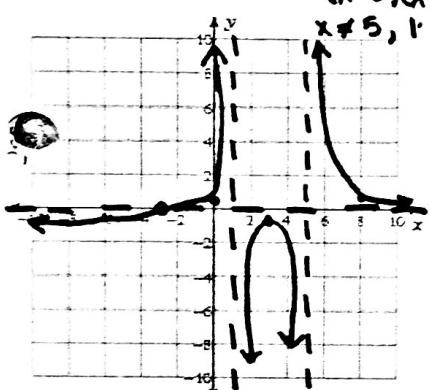
$x=-1 \quad y=\frac{4}{(-1)^2}=\frac{4}{1}=4$

$x=-2 \quad y=\frac{4}{(-2)^2}=\frac{4}{4}=1 \quad (-2,1)$

$(-1,4)$

$x=1 \quad y=\frac{4}{(1)^2}=\frac{4}{1}=4 \quad (1,4)$

b) $y = \frac{x+3}{x^2-6x+5} = \frac{x+3}{(x-5)(x-1)}$



HA: Bottom heavy, $y=0$

Holes: None, nothing cancels

VA: $x=5, x=1$

x-int: $0=x+3$

$-3=x \quad (-3,0)$

This function will cross the HA, but that is ok! (see top of page)

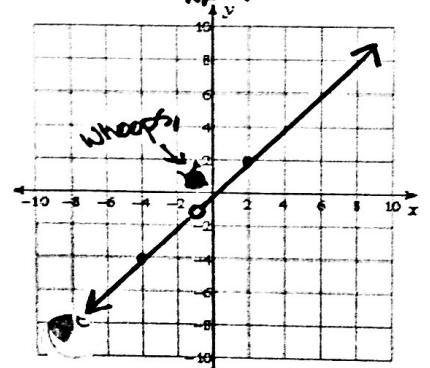
y-int: $y=\frac{0+3}{0-0+5}=\frac{3}{5} \quad (0, \frac{3}{5})$

Other: $x=-6 \quad y=\frac{-6+3}{(-6+5)(-6-1)}=\frac{-3}{(-11)(-7)}=\frac{-3}{77} \quad (-6, \frac{3}{77})$

$x=3 \quad y=\frac{3+3}{(3-5)(3-1)}=\frac{6}{(-2)(2)}=\frac{6}{-4}=-\frac{3}{2}$

$x=8 \quad y=\frac{8+3}{(8-5)(8-1)}=\frac{11}{(3)(7)}=\frac{11}{21}$

c) $y = \frac{x(x+1)}{(x+1)} = \frac{x(x+1)}{(x+1)} = x$



HA: Top heavy, none

Holes: $x=-1$ Plug into simplified function
 $y=x \rightarrow y=-1 \quad (-1, -1)$

VA: None, all cancelled

x-int: $0=x \quad (0,0)$ Plug into simplified

y-int: $y=0 \quad (0,0)$ Plug into simplified

Other: $x=2 \quad y=2 \quad (2,2)$

$x=-4 \quad y=-4 \quad (-4, -4)$