

Unit 4.3 Notes: Adding and Subtracting Rational Expressions

1. Add/Subtract fractions: Criss-cross smiley face

You can use the same rules to add and subtract rational expressions as you use to add and subtract numerical fractions:

Example: $\frac{1}{2} + \frac{3}{4} = \frac{1(4) + 3(2)}{2(4)}$
 $= \frac{4+6}{8} = \frac{10}{8} = \frac{5}{4}$

Example: $\frac{1}{3} - \frac{3}{4} = \frac{1(4) - 3(3)}{3(4)}$
 $= \frac{4-9}{12} = \frac{-5}{12}$

2. Example: add and subtract expressions with like denominators

a) $\frac{4}{3y} + \frac{7}{3y} = \frac{11}{3y}$

b) $\frac{3x}{x-2} - \frac{x}{x-2} = \frac{2x}{x-2}$

c) $\frac{5c}{2c+7} - \frac{c-28}{2c+7}$
 $= \frac{5c - (c-28)}{2c+7} = \frac{5c - c + 28}{2c+7}$
 $= \frac{4c+28}{2c+7}$

When fractions have a common denominator, add/subtract tops

3. Example: add and subtract expressions with different denominators

a) $\frac{5}{6x} + \frac{3}{2x^2} = \frac{5(2x^2) + 3(6x)}{6x(2x^2)}$
 $= \frac{10x^2 + 18x}{12x^3} = \frac{2x(5x+9)}{2x(6x^2)}$
 $= \frac{5x+9}{6x^2}$

b) $\frac{3}{7y^4} - \frac{2}{3y^2} = \frac{3(3y^2) - 2(7y^4)}{7y^4(3y^2)}$
 $= \frac{9y^2 - 14y^4}{21y^6} = \frac{y^2(9-14y^2)}{y^2(21y^4)}$
 $= \frac{9-14y^2}{21y^4} = \frac{-14y^2+9}{21y^4}$

c) $\frac{6}{5x^8} + \frac{4}{2x^2}$

d) $\frac{9}{m+2} + \frac{8}{m-7}$

Theme: Criss-cross smiley face, then factor & cancel

$$e) \frac{3}{d-1} - \frac{2}{d+2}$$

$$f) \frac{c}{3c-1} - \frac{4}{c-2}$$

$$h) \frac{x}{x^2-5x-6} - \frac{2}{x-6}$$

$$= \frac{x(x-6) + 2(x^2-5x-6)}{(x^2-5x-6)(x-6)}$$

$$= \frac{x^2-6x+2x^2-10x-12}{(x^2-5x-6)(x-6)} = \frac{3x^2-16x-12}{(x^2-5x-6)(x-6)}$$

$$= \frac{\cancel{(x-6)}(3x+2)}{\cancel{(x-6)}(x+1)(x-6)} = \boxed{\frac{3x+2}{(x+1)(x-6)}}$$

$$i) \frac{5x}{x^2-4} - \frac{3}{x^2-2x-8}$$

We don't have to distribute the bottom since we need to factor it anyway. We distribute the top so we can combine like terms then factor.

$$\begin{aligned} & \rightarrow (3x^2 - 16x - 12) \\ & 3x^2 - 18x + 2x - 12 \\ & 3x(x-6) + 2(x-6) \\ & (x-6)(3x+2) \end{aligned} \quad \begin{matrix} 36x^2 \\ -18 \hat{^} 2 \end{matrix}$$

4. A certain truck gets 25% better gas mileage when it holds no cargo than when it is fully loaded. Let m be the number of miles per gallon of gasoline the truck gets when it is fully loaded. The truck drops off a full load and returns empty. What is an expression for the number of gallons of gasoline the truck uses if he travels 80 miles in one direction?

5. A bicyclist rides 5 mi out and then rides back. His speed return is reduced by 20% because it is raining. Let r be his speed in miles per hour riding out. What is an expression that represents his total time in hours riding out and back?