$\boxed{\text{Theme: Factor \& Cancel}}$

Unit 4.2 Notes: Multiplying and Dividing Rational Expressions

Objective: To be able to multiply and divide rational expressions

1. Many problems require finding products and quotients of rational expressions.

You can multiply and divide rational expressions using the same properties you use to multiply and divide fractions.

Example: $\dfrac{1}{2} \cdot \dfrac{3}{4} = \dfrac{1 \cdot 3}{2 \cdot 4} = \dfrac{3}{8}$ Example: $\dfrac{1}{2} \div \dfrac{3}{4} = \dfrac{1}{2} \cdot \dfrac{4}{3} = \dfrac{1 \cdot 4}{2 \cdot 3} = \dfrac{4}{6} = \dfrac{2}{3}$

2. Example: What is the product? State any values that are not included in the domain.

a) $\dfrac{6}{a^2} \cdot \dfrac{-2}{a^3} = \boxed{\dfrac{-12}{a^5}}$ $\boxed{a \neq 0}$

Fractions: Multiply straight across

b) $\dfrac{x}{x-2} \cdot \dfrac{x+1}{x-3} = \boxed{\dfrac{x(x+1)}{(x-2)(x-3)}}$ $\boxed{x \neq 2, 3}$

We can leave answers in factored form

c) $\dfrac{x-7}{x} \cdot \dfrac{x+5}{x-7} = \boxed{\dfrac{x+5}{x}}$ $\boxed{x \neq 0, 7}$

check for excluded values before canceling

d) $\dfrac{b}{2} \cdot \dfrac{6}{b^3}$

e) $\dfrac{14x}{x^2+3x-10} \cdot \dfrac{x+5}{7x-21}$

$\dfrac{\overset{2}{14}x}{(x+5)(x-2)} \cdot \dfrac{x+5}{7(x-3)}$

$\boxed{\dfrac{2x}{(x-2)(x-3)}}$ $\boxed{x \neq -5, 2, 3}$

f) $\dfrac{3x^2}{x+2} \cdot \dfrac{x^2+3x+2}{x}$

$\dfrac{3x^2}{x+2} \cdot \dfrac{(x+2)(x+1)}{x}$

$\dfrac{3x(x+1)}{1} = \boxed{3x(x+1)}$ $\boxed{x \neq -2, 0}$

g) $\dfrac{2m+5}{3m-6} \cdot m^2+m-6$

g) $\dfrac{2t-14}{4t-6} \cdot 6t^2-13t+6$

Recall that $\dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{a}{b} \cdot \dfrac{d}{c}$ where $b \neq 0, d \neq 0, c \neq 0$.

Ours is not to reason why, but to flip & multiply

3. Example: Divide each pair of rational expressions and list any excluded values:

a) $\dfrac{x^2-25}{4x+28} \div \dfrac{x-5}{x^2+9x+14}$

b) $\dfrac{x}{x+y} \div \dfrac{xy}{x+y}$

c) $\dfrac{4k+8}{6k-10} \div \dfrac{k^2+6k+8}{9k-15}$

$\dfrac{x^2-25}{4x+28} \cdot \dfrac{x^2+9x+14}{x-5}$

$* \dfrac{(x+5)(x-5)}{4(x+7)} \cdot \dfrac{(x+7)(x+2)}{x-5}$ $x \neq -7, -2$

$x \neq -7$ $x \neq 5$

$\boxed{\dfrac{(x+5)(x+2)}{4}}$ $\boxed{x \neq -7, 5, -2}$

d) $\dfrac{x-1}{x+4} \div \dfrac{x+3}{x+4}$

$\dfrac{x-1}{x+4} \cdot \dfrac{x+4}{x+3}$ $x \neq -4$

$x \neq -4$ $x \neq -3$

$\boxed{\dfrac{x-1}{x+3}}$ $\boxed{x \neq -4, -3}$

e) $\dfrac{3x^2-12x}{5x} \div x^2 - 3x - 4$ f) $\dfrac{z^2-2z+1}{z^2+2} \div (z-1)$

* Excluded Values

Because we are dealing with two separate rational expressions, we need to check for excluded values on both of them before we divide. But since we combine them, we need to check the excluded values of the finished product.

Translation: When dividing, flip, factor, check excluded values of both bottoms & top of the right fraction.

A **Complex Fraction** is a fraction that contains one or more fractions in its numerator, in its denominator, or in both. You can simplify a complex fraction by dividing its numerator by its denominator.

Any complex fraction of the form $\dfrac{\frac{a}{b}}{\frac{c}{d}}$ can be expressed as $\dfrac{a}{b} \div \dfrac{c}{d}$

4.   Example: Simplify each Complex Fraction

a)
$$\dfrac{\frac{1}{x-2}}{\frac{x+3}{x^2-4}}$$

b)
$$\dfrac{\frac{g+2}{3g-1}}{\frac{g^2+2g}{6g+2}}$$

c)
$$\dfrac{\frac{c+4}{c^2+5c+6}}{\frac{3c^2+12c}{2c^2+5c-3}}$$

a)
$$\dfrac{1}{x-2} \div \dfrac{x+3}{x^2-4}$$

$$\dfrac{1}{x-2} \cdot \dfrac{x^2-4}{x+3}$$

$$\dfrac{1}{x-2} \cdot \dfrac{(x+2)(x-2)}{x+3} \quad x \neq -2, 2$$

$$x \neq 2 \qquad\qquad x \neq -3$$

$$\boxed{\dfrac{x+2}{x-3}} \quad \boxed{x \neq 2, -3, -2}$$

c)
$$\dfrac{c+4}{c^2+5c+6} \div \dfrac{3c^2+12c}{2c^2+5c-3}$$

$$\dfrac{c+4}{c^2+5c+6} \cdot \dfrac{2c^2+5c-3}{3c^2+12c}$$

$$\dfrac{c+4}{(c+2)(c+3)} \cdot \dfrac{(c+3)(2c-1)}{3c(c+4)}$$

$$c \neq -2, -3 \qquad c \neq 0, -4$$

$$(\text{Top}) \ c \neq -3, \tfrac{1}{2}$$

$$\boxed{\dfrac{2c-1}{3c(c+2)}} \quad \boxed{\begin{array}{c} c \neq -2, -3, 0, \\ -4, -\tfrac{1}{2} \end{array}}$$

$$(2c^2 + 5c - 3) \qquad \dfrac{-6c^2}{6} \ {}^{-1}$$

$$2c^2 + 6c - 1c - 3$$

$$2c(c+3) - 1(c+3)$$

$$(c+3)(2c-1)$$