3.4 Solve Polynomial Equati

Objective: Solve any polynomial equation, regardless of degr

• How many solutions will you have?
• Does it factor?
- 4 terms: factor by grouping
- GCF
- difference of squares
• If not, do you need a placeholder for synthetic division?
- break down to quadratic & solve using factoring or quadratic formula

Now we get to put together everything we've learned in this
Essentially, the goal is to divide off linear factors (using the Ra
Theorem) until you are left with a quadratic that you can eith
use the discriminant to decide how to handle the quadratic y

If you start with a cubic (3rd degree polynomial), there is one
polynomials can be factored by grouping.

Example 1. Solve each of the following cubic equations by factoring by grouping.

a) $x^3 - 3x^2 - 4x + 12 = 0$ 4 terms: Difference of squares

$x^2(x-3) - 4(x-3) = 0$

$(x-3)(x^2-4) = 0$ Difference of squares

$x=3$ $(x+2)(x-2)$

$x=-2$ $x=2$

$\boxed{x = 2, -2, 3}$

b) $x^3 + 5x^2 - 5x - 25 = 0$

$x^2(x+5) - 5(x+5) = 0$

$(x^2-5)(x+5) = 0$

$x^2-5=0$ $x=-5$

$x^2 = 5$

$x = \pm\sqrt{5}$ $\boxed{x = -5, \sqrt{5}, -\sqrt{5}}$

* If you don't feel comfortable factoring, set factor equal to 0 and solve

For polynomials that cannot be easily factored by grouping, we will need to use the tried-and-true combination of the Rational Root Theorem and the Remainder Theorem to work our equation down to a smaller degree.

1. Check if there is anything that can be factored out of every term.
    a) is there a number that divides evenly into each coefficient?
    b) does every term have an x on it?
2. Use the Rational Root Theorem to list the possible rational roots of the polynomial.
3. Use the Remainder theorem to see if each possible root is an actual root
4. When left with a quadratic, use the appropriate method to solve the quadratic

Example 2. Find all roots of the following polynomial Equations.

a) $x^4 - 5x^3 + 7x^2 - 3x = 0$ • 4 solutions • GCF: x • No placeholders

$x(x^3 - 5x^2 + 7x - 3) = 0$

$\boxed{x=0}$ Last(3): 1, 3 First(1): 1 $\pm 1, \pm 3$

$$\begin{array}{r|rrrr} 1 & 1 & -5 & 7 & -3 \\ & & 1 & -4 & 3 \\ \hline & 1 & -4 & 3 & \boxed{0} \\ & x^2 & x & c & \end{array}$$

$x^2 - 4x + 3$ $3x^2$ -3 -1

$(x-3)(x-1)$

$\boxed{x=3}$ $\boxed{x=1}$

$\boxed{x = 0, 1 \text{ mult. } 2, 3}$

b) $x^3 - 9x^2 - 21x = 11$

c) $3x^3 + 2x^2 - 3x - 2 = 0$

When you break down to a quadratic and it doesn't factor, you need to use quadratic formula.

d) $x^4 - 5x^2 + 4 = 0$ • 4 solutions • "Doesn't factor" • 2 placeholders

Last(4): 1, 2, 4
First(1): 1 $\pm 1, \pm 2, \pm 4$

$$\begin{array}{r|rrrrr} 1 & 1 & 0 & -5 & 0 & 4 \\ & & 1 & 1 & -4 & -4 \\ \hline & 1 & 1 & -4 & -4 & \boxed{0} \\ & \cancel{x^3} & \cancel{x^2} & \cancel{x} & \cancel{c} & \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & 1 & -4 & -4 \\ & & -1 & 0 & 4 \\ \hline & 1 & 0 & -4 & \boxed{0} \\ & x^2 & x & c & \end{array}$$

$x^2 - 4$
$(x+2)(x-2)$
$x=-2$ $x=2$

If it's not a quadratic, do synthetic division with the new coefficients

$\boxed{x = 1, -1, 2, -2}$