Unit 3.2: Factoring Quadratics with a Leading Coefficient of 1

4 terms: Factoring by grouping

Factor the following expressions.

a. $2x^3 - x^2 + 4x - 2$

$x^2(2x-1)+2(2x-1)$

$\boxed{(2x-1)(x^2+2)}$

b. $x^3 - 5x^2 - 3x + 15$

$x^2(x-5)-3(x-5)$

$\boxed{(x-5)(x^2-3)}$

c. $2wx + 10w + 7x + 35$

d. $6x^4 - 24x^3 - 5x - 45$

The rest of the unit will be focused on factoring quadratics in **standard form**, which is $\underline{ax^2+bx+c}$.

When factoring trinomials, we are basically trying to create a situation where we can use factoring by grouping. To do this, our main focus will be to split the middle term into two terms.

Factor each expression.

Steps
1 i. Find two numbers that multiply to $a \cdot c$ (the outside) and add to b (the middle).
2 ii. Split the middle term into two pieces using the numbers from part (i).
3 iii. Factor by Grouping

a. $n^2 - 7n + 6$

1 $6n^2$

$-6n \quad -1n$

2 $n^2-6n-1n+6$

3 $n(n-6)-1(n-6)$ $\boxed{(n-6)(n-1)}$

b. $m^2 - m - 56$

$-56m^2$

$-8m \quad 7m$

$m^2-8m+7m-56$

$m(m-8)+7(m-8)$

$\boxed{(m-8)(m+7)}$

c. $r^2 - 16r + 64$

$64r^2$

$-8r \quad -8r$

$r^2-8r-8r+64$

$r(r-8)-8(r-8)$

$(r-8)(r-8) = \boxed{(r-8)^2}$

d. $v^2 - 9$

v^2+0v-9

$-9v^2$

$-3v \quad 3v$

* Missing middle term: replace with 0

$v^2-3v+3v-9$

$v(v-3)+3(v-3)$

$\boxed{(v-3)(v+3)}$

e. $x^2 - 15x + 50$

f. $n^2 + 11n + 30$

Look back on all of the examples that you did. What patterns do you notice?

The numbers that multiply to the middle term always end up as the numbers in the parentheses.

When a=1, this pattern happens every time! This means that when a=1, we can use a shortcut to factor.

* Can only use shortcut with no coefficient in front of x^2

3) Factor each expression.

a. $x^2 + 7x + 6$ $6x^2$ $6x$ $1x$

$\boxed{(x+6)(x+1)}$

b. $x^2 + 4x - 32$ $-32x^2$ $8x$ $-4x$

$\boxed{(x+8)(x-4)}$

Now that we've had practice when a=1, let's combine this with a greatest common factor.

* Take out GCF first

4) Factor each expression.

a. $2v^2 + 18v + 40$ $20v^2$

$2(v^2+9v+20)$ $5v$ $4v$

$\boxed{2(v+5)(v+4)}$

b. $3x^2 - 12x - 36$ $-12x^2$

$3(x^2-4x-12)$ $-6x$ $2x$

$3(x-6)(x+2)$

c. $2x^2 + 16x - 66$ $-33x^2$

$2(x^2+8x-33)$ $11x$ $-3x$

$\boxed{2(x+11)(x-3)}$

d. $4y^2 + 20y - 56$ $-14y^2$

$4(y^2+5y-14)$ $7y$ $-2y$

$\boxed{4(y+7)(y-2)}$

e. $3a^3 - 6a^2 + 240a$ $80a^2$

$3a(a^2-2a+80)$ $-10a$ $8a$

$\boxed{3a(a-10)(a+8)}$

f. $12x^2 - 48$

$12(x^2-4)$ $-4x^2$

$12(x^2+0x-4)$ $2x$ $-2x$

$\boxed{12(x+2)(x-2)}$

Let's look at some application problems of factoring a quadratic trinomial:

5) A square has an $\boxed{\text{area}}$ of $x^2 + 10x + 25$. Write an expression in terms of x for the possible length and width of the square.

$A = l \cdot w$

$x^2+10x+25$ $25x^2$ $5x$ $5x$

$\boxed{(x+5)(x+5)}$

6) The Johnsons are putting a fence in their backyard, but are very picky about the ratio of the fence dimensions. They want to make sure that the area of the lawn is always represented by x^2 $-9x + 20$. What expressions could represent the dimensions of their fence?

$x^2-9x+20$ $20x^2$ $-5x$ $-4x$

$\boxed{(x-5)(x-4)}$