

Unit 3.1: Greatest Common Factor

Simplify.

a. $2(5x^2 - 3x + 7)$

$$10x^2 - 6x + 14$$

b. $x(x - 4)$

$$x^2 - 4x$$

c. $3x^2(x - 2x + 8)$

$$3x^3 - 6x^3 + 24x^2$$

$$\boxed{-3x^3 + 24x^2}$$

The process which you just used is called distribution. But sometimes we are given a problem where they want us to "undo" the distribution that has occurred. This process is called factoring.

There are many different types of factoring based on how much you would like to "undistribute" an expression. Today we will be focusing on taking out the **greatest common factor (GCF)**. **GCF: What is the biggest thing each term is divisible by?**

2) Factor the greatest common factor out of each expression.

a. $10x^2 - 6x + 14$

$$2(5x^2 - 3x + 7)$$

b. $15x^3 - 5x^2 - 20x + 5$

$$5(3x^3 - x^2 - 4x + 1)$$

c. $27x + 30$

$$3(9x + 10)$$

d. $-x^2 + 3x - 4$

$$-(x^2 - 3x + 4)$$

* Factor out negative when first term is negative

Make sure everything is in standard form before starting

3) Factor the greatest common factor out of each expression.

a. $x^2 - 4x$

$$x(x - 4)$$

b. $3b^3 + 2b^2 - 4b$

$$b(3b^2 + 2b - 4)$$

c. $8m^6 + 2m^8 - 7m^4$ standard form first

$$2m^8 + 8m^6 - 7m^4$$

$$\boxed{m^4(2m^4 + 8m^2 - 7)}$$

* Look for smallest exponent & factor it out

d. $5y^6 + y^3 - y$

$$y(5y^5 + y^2 - 1)$$

Reminder:

$$\frac{8m^6}{m^4} = \frac{8 \cdot m \cdot m \cdot m \cdot m \cdot m \cdot m}{m \cdot m \cdot m \cdot m} = 8m^2$$

4) Factor the greatest common factor out of each expression

a. $8m^8 - 2m^6$

$$2m^6(4m^2 - 1)$$

b. $-6x^5 + 2x^4 - 10x^3$

$$-2x^3(3x^2 - 2x + 5)$$

c. $3x^4 - 6x^3 + 24x^2$

$$3x^2(x^2 - 2x + 8)$$

d. $-7y^4 + 21y^7 + 7y^5$

$$7y^4(3y^3 + y - 1)$$

$$\boxed{7y^4(3y^3 + y - 1)}$$

GCF can be a number, a variable, or a combination of both.

5) Describe what the greatest common factor is in your own words:

Factoring by Grouping

~~4~~ 4 terms

Idea: Split into groups and factor out GCF from both groups

If you are given four terms and are asked to factor, you may want to consider factoring by grouping. Factoring by grouping creates smaller groups within the problem.

Before you start any kind of factoring, you should always look for a greatest common factor.

Steps to factoring by grouping:

- ① i. Group the first two terms together and the last two together.
- ② ii. Factor the GCF from each of the two groups. Notice that what is left inside the parenthesis is a perfect match. This is now the GCF of the two remaining terms.
- ③ iii. Factor the "match" out of the two remaining terms. Your polynomial is now factored!

① a) $4x^2 + 20xy - 3xy - 15y$

② $4x(x+5) - 3y(x+5)$

③ $(x+5)(4x-3y)$

b) $3x^3 - 6x^2 + 15x - 30$

c) $4x^3 + 8x^2 + x + 2$

$4x^2(x+2) + 1(x+2)$

$(x+2)(4x^2+1)$

e) $x^2y - 3x^2 - 8y + 24$

* If there is no GCF, factor out a 1

d) $3xy - 21y + 5x - 35$

$3y(x-7) + 5(x-7)$

$(x-7)(3y+5)$

f) $8x^3 + 6x^2 - 24x - 18$