Unit 2.2: Roots and Remainders

Ex: $\dfrac{x^3-2x^2+7x-1}{x+5}$

Long Division

$$\require{enclose} \begin{array}{r} x^2-7x+42 \\ x+5 \enclose{longdiv}{x^3-2x^2+7x-1} \\ -x^3+5x^2 \\ \hline -7x^2+7x \\ +7x^2+35x \\ \hline 42x-1 \\ -42x+210 \\ \hline -211 \end{array}$$

$\boxed{x^2-7x+42\ \frac{-211}{x+5}}$

Synthetic Division

$$\begin{array}{r|rrrr} -5 & 1 & -2 & 7 & -1 \\ & & -5 & 35 & -210 \\ \hline & 1 & -7 & 42 & -211 \\ & x^2 & x & c & \end{array}$$

$\boxed{x^2-7x+42\ -\frac{211}{x+5}}$

We like synthetic division for multiple reason. The main reason is because it is faster than long division. Because of this, we are able to find out other things about polynomials in a lot less steps than if we had used long division.

Let's review synthetic division:

*Need placeholders when we don't have all powers of x

Synthetic Division

1) Put the zero of what you're dividing by on outside & coefficients of polynomial on inside *Placeholders
2) Drop first coefficient
3) Multiply by the zero & write in second row
4) Add down
5) Repeat steps 3&4 until done

Ex: $\dfrac{x^3+8x^2-3x+1}{x-3}$ ← The zero of x-3 is 3 (since 3-3=0)

$$\begin{array}{r|rrrr} 3 & 1 & 8 & -3 & 1 \\ & & 3 & 33 & 90 \\ \hline & 1 & 11 & 30 & 91 \\ & x^2 & x & c & \end{array}$$

Remainder

$\boxed{x^2+11x+30+\frac{91}{x-3}}$

We can only use synthetic division when dividing by a linear binomial (ex: x-3, 2x+1, x+5)

If this condition is not met, we have to use long division. In general, though, we will only use long division when the instructions tell us to do so.

The rest of the lesson is going to be focused on the benefits of synthetic division.

1) Find the remainder of the polynomial.

a. $\dfrac{x^4-18x^3+88x^2-61x-20}{x-9}$

$$\begin{array}{r|rrrrr} 9 & 1 & -18 & 88 & -61 & -20 \\ & & 9 & -81 & 63 & 18 \\ \hline & 1 & -9 & 7 & 2 & \boxed{-2} \end{array}$$

b. $\dfrac{x^4+16x^3+56x^2-58x+45}{x-8}$

$$\begin{array}{r|rrrrr} 8 & 1 & 16 & 56 & -58 & 45 \\ & & 8 & 192 & 1984 & 15408 \\ \hline & 1 & 24 & 248 & 1926 & \boxed{15453} \end{array}$$

c. $\dfrac{x^3-12x^2+21x+7}{x-2}$

$$\begin{array}{r|rrrr} 2 & 1 & -12 & 21 & 7 \\ & & 2 & -20 & 2 \\ \hline & 1 & -10 & 1 & \boxed{9} \end{array}$$

*d. $\dfrac{x^4-3x^3+2x-12}{x-3}$ $0x^2$

* Need placeholder since x^2 is missing

$$\begin{array}{r|rrrrr} 3 & 1 & -3 & 0 & 2 & -12 \\ & & 3 & 0 & 0 & 6 \\ \hline & 1 & 0 & 0 & 2 & \boxed{-6} \end{array}$$

Sometimes you will be asked if a factor is a root. If the factor is a root, then it will divide nicely into the polynomial. So basically you are looking for a remainder of 0.

2) Determine if the factor is a root of the polynomial. If so, write the polynomial as a product of two factors.

a. $\dfrac{x^4-x^3+9x^2-9}{x-1}$ $0x$

* Need placeholder since x is missing

$$\begin{array}{r|rrrrr} 1 & 1 & -1 & 9 & 0 & -9 \\ & & 1 & 0 & 9 & 9 \\ \hline & 1 & 0 & 9 & 9 & \boxed{0} \\ & x^3 & x^2 & x & c & \end{array}$$

$\boxed{\text{Yes, } (x-1)(x^3+9x+9)}$

b. $\dfrac{x^3-3x^2-8x+28}{x-2}$

Basically multiply what you divided by by your result

$$\begin{array}{r|rrrr} 2 & 1 & -3 & -8 & 28 \\ & & 2 & -2 & -20 \\ \hline & 1 & -1 & -10 & \boxed{8} \end{array}$$

$\boxed{\text{Not a root}}$

c. $\dfrac{7x^4-14x^3+6}{x-2}$

$$\begin{array}{r|rrrrr} 2 & 7 & -14 & 0 & 0 & 6 \\ & & 14 & 0 & 0 & 0 \\ \hline & 7 & 0 & 0 & 0 & \boxed{6} \end{array}$$

$\boxed{\text{Not a root}}$

d. $\dfrac{x^3-3x^2+10}{x-3}$

$$\begin{array}{r|rrrr} 3 & 1 & -3 & 0 & 10 \\ & & 3 & 0 & 0 \\ \hline & 1 & 0 & 0 & \boxed{10} \end{array}$$

$\boxed{\text{Not a root}}$

If you are asked to find all roots of a polynomial, break down to a quadratic, then factor.

3) Find all roots of the polynomial. Then write the polynomial as a product of all its factors.

a. $\dfrac{x^3-9x^2+17x-21}{x-7}$

b. $\dfrac{9x^3-49x^2+20x}{x-5}$

$$\begin{array}{r|rrrr} 5 & 9 & -49 & 20 & 0 \\ & & 45 & -20 & 0 \\ \hline & 9 & -4 & 0 & \boxed{0} \\ & x^2 & x & c & \end{array}$$

Take out GCF $9x^2-4x$

$x(9x-4)$

$\boxed{x(9x-4)(x-5)}$

c. $\dfrac{2x^3+3x^2-39x-20}{x-4}$

$$\begin{array}{r|rrrr} 4 & 2 & 3 & -39 & -20 \\ & & 8 & 44 & 20 \\ \hline & 2 & 11 & 5 & \boxed{0} \\ & x^2 & x & c & \end{array}$$

$2x^2+11x+5$ $\quad 10x^2$ $\quad 10x \wedge 1x$

$2x^2+10x+1x+5$

$2x(x+5)+1(x+5)$

$(x+5)(2x+1)$

$\boxed{(x+5)(2x+1)(x-4)}$

d. $\dfrac{15x^3-106x^2+x+42}{x-7}$

$$\begin{array}{r|rrrr} 7 & 15 & -106 & 1 & 42 \\ & & 105 & -7 & 42 \\ \hline & 15 & -1 & -6 & \boxed{0} \\ & x^2 & x & c & \end{array}$$

$15x^2-x+6$ $\quad 90x^2$ $\quad 9x \wedge -10x$

$15x^2+9x-10x+6$

$3x(5x+3)-2(5x+3)$

$(5x+3)(3x-2)$

$\boxed{(5x+3)(3x-2)(x-7)}$

Application questions:

4) If the polynomial $x^3+6x^2+11x+6$ expresses the volume, in cubic inches, of the box, and the width is $(x+1)$ in., what are the dimensions of the box?

Use synthetic division to find other dimensions.

$$\begin{array}{r|rrrr} -1 & 1 & 6 & 11 & 6 \\ & & -1 & -5 & -6 \\ \hline & 1 & 5 & 6 & \boxed{0} \\ & x^2 & x & c & \end{array}$$

x^2+5x+6 $\quad 6x^2$ $\quad 2x \wedge 3x$

$(x+2)(x+3)$

$\boxed{(x+2)\text{ in} \; (x+3)\text{ in} \; (x+1)\text{ in}}$

5) The volume, in cubic inches, of the decorative box shown can be expressed as the product of the lengths of its sides as $V(x) = x^3-4x^2-9x+36$. What linear expressions represent the length and height of the box?