Unit 2.1 Notes: Polynomial Operations

Adding/Subtracting: Combine like terms

Like terms: Same variable, same exponent

Standard form: In order from highest to lowest exponent

1) Simplify each expression and write in standard form.

a. $(7x^3 - 4x^2 + 5x) + (3x^3 - 11x + 2)$

$\boxed{10x^2 - 4x^2 - 6x + 2}$

b. $x^2 - 7x + 12) + (-5x^2 + 2x)$

c. $(x^4 - 2x^2 + 3x) - (4x^4 - 9x^2 + 7)$

$x^4 - 2x^2 + 3x \quad -4x^4 + 9x^2 - 7$

$\boxed{-3x^4 - 7x^2 + 3x - 7}$

d. $x^2 + 7x - 8) - (7x^3 + 2x^3 + 3x)$

When subtracting, distribut Negative through

Multiplying:

2) Simplify each expression and write in standard form.

a. $(2x + 3y)(7x - 4y)$

$14x^2 - 8xy$
$+ 21xy - 12y^2$

$\boxed{14x^2 + 13xy - 12y^2}$

b. $(x - 5)(4x^2 + 9x - 1)$

$4x^3 + 9x^2 - x$
$-20x^2 - 45x + 5$

$\boxed{4x^3 - 11x^2 - 46x + 5}$

c. $(x^2 + 2)(x + 3)(2x + 1)$

$(x^3 + 3x^2 + 2x + 6)(2x + 1)$

Pick two, then continue

$2x^4 + 6x^3 + 4x^2 + 12x$
$x^3 + 3x^2 + 2x + 6$

$\boxed{2x^4 + 7x^3 + 7x^2 + 14x + 6}$

Dividing

Let's review long division with $\frac{4572}{3}$.

$$\begin{array}{r} 1524 \\ 3\overline{)4572} \\ -3 \\ \hline 15 \\ -15 \\ \hline 0 \end{array}$$

-6, 12

Just like we can use long division to divide numbers, we can also use long division to divide polynomials. If the remainder is 0, then you have found a factor of the polynomial.

Long Division Steps

1) Write denominator outside divide sign & numerator inside
2) What do you need to multiply first term by to get inside stuff?
   • write on top
3) Distribute through & write below
4) Subtract
5) Repeat

3) Use long division to divide each polynomial.

a. $\frac{x^3+7x^2+15x+9}{x+1} = \boxed{x^2+6x+9}$

$$\begin{array}{r} x^2+6x+9 \\ x+1\,\overline{)\,x^3+7x^2+15x+9} \\ -\;x^3-x^2 \\ \hline 6x^2+15x \\ -\;6x^2-6x \\ \hline 9x+9 \\ 9x+9 \\ \hline 0 \end{array}$$

b. $\frac{2x^3-7x^2-7x+13}{x-4} = \boxed{2x^2+x-3+\frac{1}{x-4}}$

$$\begin{array}{r} 2x^2+x-3 \\ x-4\,\overline{)\,2x^3-7x^2-7x+13} \\ -\;2x^3+8x^2 \\ \hline x^2-7x \\ -x^2+4x \\ \hline -3x+13 \\ -3x+12 \\ \hline 1 \end{array}$$

c. Is x^4-1 a factor of $P(x)=x^5+5x^4-x-5$? If so, write $P(x)$ as a product of two factors.

$$\begin{array}{r} x+5 \\ x^4-1\,\overline{)\,x^5+5x^4+0x^3+0x^2-x-5} \\ x^5 \qquad\qquad\qquad +x \quad \\ \hline 5x^4 \qquad\qquad\qquad -5 \\ 5x^4 \qquad\qquad\qquad -5 \\ \hline 0 \end{array}$$

* Fill in missing terms with 0

x^4-1 is a factor

$\boxed{(x^4-1)(x+5)}$

Recap: When you are asked to determine if something is a factor of an expression, you use __division__ and are looking for __a remainder of 0__.

As fun as long division is, there is a faster way called **synthetic division**. It basically allows us to deal strictly with the coefficients and worry about the variables later.

We can only use synthetic division when __dividing by a linear binomial.__

Ex: $\frac{x^3-3x^2-5x-25}{x-5} = \boxed{x^2+2x+5}$

$$\begin{array}{r|rrrr} 5 & 1 & -3 & -5 & -25 \\ & & 5 & 10 & 25 \\ \hline & 1 & 2 & 5 & \boxed{0} \\ & x^2 & x & c & \end{array}$$

Remainder

Steps
1) Put zero of factor on outside of divide sign & coefficients of polynomial on inside
   * Placeholders if needed
2) Drop down first coefficient
3) Multiply number on bottom by the zero & write in second row
4) Add down
5) Repeat steps 3 & 4

4) Use synthetic division to divide the polynomial.

a. $(6x^2 - 8x - 2) \div (x - 1)$

$x = 1$

$$\begin{array}{r|rrr} 1 & 6 & -8 & -2 \\ & & 6 & -2 \\ \hline & 6 & -2 & \boxed{-4} \end{array}$$

$x \quad c$

$\boxed{6x - 2 + \frac{-4}{x-1}}$

b. $(2x^3 + 14x^2 - 58x) \div (x + 10)$

$$\begin{array}{r|rrrr} -10 & 2 & 14 & -58 & 0 \\ & & -20 & 60 & -20 \\ \hline & 2 & -6 & 2 & \boxed{-20} \end{array}$$

$\boxed{2x^2 - 6x + 20 - \frac{20}{x+10}}$

c. $(x^3 + 5x^2 - x - 5) \div (x + 5)$

$$\begin{array}{r|rrrr} -5 & 1 & 5 & -1 & -5 \\ & & -5 & 0 & 5 \\ \hline & 1 & 0 & -1 & \boxed{0} \end{array}$$

$x^2 \quad x \quad c$

$\boxed{x^2 - 1}$

$x^2 + 0x - 1$

d. $(x^3 - 4x^2 - 25) \div (x - 5)$

$$\begin{array}{r|rrrr} 5 & 1 & -4 & 0 & 25 \\ & & 5 & 5 & 25 \\ \hline & 1 & 1 & 5 & \boxed{50} \end{array}$$

$x^2 \quad x \quad c$

$\boxed{x^2 + x + 5 + \frac{50}{x-5}}$

5) If the polynomial $x^3 + 6x^2 + 11x + 6$ expresses the volume, in cubic inches, of the box, and the width is $(x + 1)$ in., what are the dimensions of the box?

6) The volume, in cubic inches, of the decorative box shown can be expressed as the product of the lengths of its sides as $V(x) = x^3 - 4x^2 - 9x + 36$. What linear expressions represent the length and height of the box?