11.4 Binomial Theorem

$(x+y)^0$	1
$(x+y)^1$	$x + y$
$(x+y)^2$	$x^2 + 2xy + y^2$
$(x+y)^3$	$x^3 + 3x^2y + 3xy^2 + y^3$
$(x+y)^4$	$x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

What patterns do you notice with the exponents of each variable? What patterns do you notice with the coefficients of each term?

* First term exponent starts high and go down, second term exponent starts at 0 and goes up

Pascal's Triangle gives us the coefficients of expanded binomials.

Row # represents exponent of the binomial

To find next row, add two numbers that are above it

Row 0 1
Row 1 1 1
Row 2 1 2 1
Row 3 1 3 3 1
Row 4 1 4 6 4 1
Row 5 1 5 10 10 5 1
Row 6 1 6 15 20 15 6 1
Row 7 1 7 21 35 35 21 7 1
Row 8 1 8 28 56 70 56 28 8 1
Row 9 1 9 36 84 126 126 84 36 9 1
Row 10 1 10 45 120 210 252 210 120 45 10 1

1) Expand each polynomial. *Use Pascal's triangle to get coefficients & apply pattern of exponents

a) $(a+b)^6$ Row 6

$1a^6b^0 + 6a^5b^1 + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6a^1b^5 + 1a^0b^6$

$a^6 + 6a^5 + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$

b) $(x+3)^4$ Row 4

$1x^4 3^0 + 4x^3 3^1 + 6x^2 3^2 + 4x^1 3^3 + 1x^0 3^4$

Simply by multiply all numbers in each term

$4 \cdot 3^1 \quad 6 \cdot 3^2 \quad 4 \cdot 3^3 \quad 3^4$

$x^4 + 12x^3 + 54x^2 + 108x + 81$

c) $(x-5)^3$

d) $(x+2)^8$

*Remember: exponent applies to everything in parentheses

e) $(3x-2)^4$ Row 4

$1(3x)^4(-2)^0 + 4(3x)^3(-2)^1 + 6(3x)^2(-2)^2 + 4(3x)^1(-2)^3 + 1(3x)^0(-2)^4$

$3^4 x^4 \quad 4\cdot 3^3 x^3(-2) \quad 6\cdot 3^2 x^2\cdot(-2)^2 \quad 4\cdot 3\,(-2)^3$

$81x^4 - 216x^3 + 216x^2 - 96x + 16$

f) $(4a+7)^6$

2) Find the specified term for each.

a) 3rd term for $(x-2)^5$ Row 5

$1x^5(-2)^0 + 5x^4(-2)^1 + \underline{10x^3(-2)^2}$ 3rd term

$10\cdot(-2)^2$

$40x^3$

b) 5th term for $(x+2)^6$

c) 3rd term for $(2x-1)^3$

d) 4th term for $(3x+5)^7$